Knowledge Organiser: Mathematics
Year 12 – Summer Term 1

Suggested websites: TL Maths and Maths Watch

Key point

The general equation of an **exponential function** is $y = a^x$ where a is a positive constant.

Key point

$x = a^n$

$n = \log_a x$

n is the log of x to base a

Unit 8: Exponentials and logarithms (PURE)

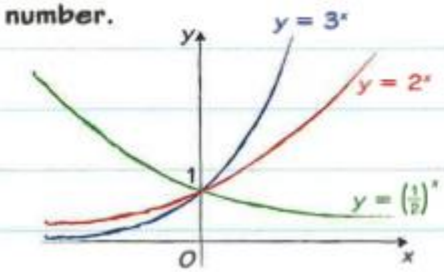
- 8a. Exponential functions and natural logarithms

Key Vocabulary

Exponential, exponent, power, logarithm, base, initial, rate of change, compound interest

Exponential functions

You need to be able to sketch the graph of $y = a^x$. You can only sketch this graph when a is a **positive number.**



$y = a^x$

- ✓ Passes through (0, 1).
- ✓ $y = 0$ is an **asymptote.**
- ✓ If $a > 1$ graph curves **upwards.**
- ✓ If $0 < a < 1$ graph curves **downwards.**

Logarithms

Logarithms (or **logs**) are a way of writing facts about **powers.** These two statements mean the same thing:

You say 'log to the base a of b equals x'.

$\log_a b = x \longleftrightarrow a^x = b$

a is the **base** of the logarithm.

For example: $\log_3 9 = 2 \longleftrightarrow 3^2 = 9$

Remembering the order

The key to being confident in log questions is remembering the **basic definition.**

Start at the **base,** and work in a **circle.**

$\log_a b = x$

... to get b

Raise a to the power x...

Laws of logarithms

Learn these four key laws for manipulating expressions involving logs. These laws all work for logarithms with **the same base.**

1. $\log_a x + \log_a y = \log_a (xy)$

   $\log_4 8 + \log_4 2 = \log_4 16 = 2$ (since $4^2 = 16$)

2. $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

   $\log_9 18 - \log_9 6 = \log_9 3 = \frac{1}{2}$ (since $9^{\frac{1}{2}} = 3$)

3. $\log_a \left(\frac{1}{x}\right) = -\log_a x$

4. $\log_a (x^n) = n \log_a x$

   $\log_5 (25^3) = 3 \log_5 25 = 3 \times 2 = 6$

Worked example

Express $3\log_a 2 + \log_a 10$ as a single logarithm to base a. **(3 marks)**

$3\log_a 2 + \log_a 10 = \log_a(2^3) + \log_a 10$

$= \log_a(2^3 \times 10)$

$= \log_a 80$

Use law 4 to write $3\log_a 2$ as $\log_a(2^3)$, then use law 1 to combine the two logarithms.

Equations with logs

If you see an equation involving **logarithms** in your exam, you will probably need to rearrange it using the **laws of logarithms,** which are covered on page 48.

Two steps to solving log equations

Follow these two steps to solve most log equations in your exam:

1. Group the log terms on one side, then use the laws of logs on page 48 to write them as a **single algorithm.**
2. Rewrite $\log_a f(x) = k$ as $f(x) = a^k$ and solve the equation to find x.

Undefined logs

The value $\log_a b$ is **only defined** for $b > 0$. You can't calculate $\log_a 0$ or the log of any negative number. If an equation contains $\log_a x$ or $\log_a kx$ then **ignore** any solutions where $x \leqslant 0$.

If there are solutions to ignore in an exam question, you will usually be given a **range** of possible values for x.

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Key point
As you lower the significance level, you need more evidence to reject the null hypothesis and you lower the chance of making an incorrect conclusion.

Unit 5b: Statistical hypothesis testing (Stats)

5b. Carry out hypothesis tests involving the binomial distribution

Key Vocabulary

. Hypotheses, significance level, one-tailed test, two-tailed test, test statistic, null hypothesis, alternative hypothesis, critical value, critical region, acceptance region, p-value, binomial model, accept, reject, sample, inference.

Hypothesis testing

You need to be able to carry out a hypothesis test for the probability, p, in a binomial distribution. Follow these steps to carry out a hypothesis test.

1. Model the test statistic and define null (H_0) and alternative (H_1) hypotheses.
2. Assume H_0 is true and calculate the probabilityof the observed value (or a greater / lesser value) occurring
3. Compare this probability with a given significance level and write a conclusion stating whether H_0 is accepted or rejected.

Key point
The **alternative hypothesis** contradicts the null hypothesis.
It might state that the true value is greater than, less than or simply different to the value stated in the null hypothesis.

Key point
The **null hypothesis** is your starting assumption.
It states that a parameter, such as the probability, takes a certain value, and you assume this to be true in your calculations.

Strategy

To interpret the result of a hypothesis test

1. Identify the critical region and draw a diagram if it helps.
2. If the value from the sample lies in the critical region then you reject the null hypothesis. If the value does not lie in the critical region then you accept the null hypothesis.
3. End with a conclusion that relates back to the situation described in the question.

Example 1

A shop makes this claim: '85% of our customers are satisfied with our service.'

Let p be the probability that a customer, chosen at random, is satisfied.

Write the null hypothesis and the alternative hypothesis in these cases.

a The claim is believed to be an overestimate.
b The claim is believed to be an underestimate.
c The claim is believed to be incorrect.

a $H_0: p = 0.85$ and $H_1: p < 0.85$ — H_1 expresses the alternative suggestion.

b $H_0: p = 0.85$ and $H_1: p > 0.85$ — If H_0 is an underestimate the true value must be higher.

c $H_0: p = 0.85$ and $H_1: p \neq 0.85$ — The true value of p could be more or less than 0.85

In a discrete distribution such as the binomial distribution, the probability of incorrectly rejecting H_0 is the probability represented in the critical region and is therefore **less than or equal to** the significance level.

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Key point

$$v = \frac{ds}{dt} \quad \text{or} \quad v = \dot{s}$$

Key point

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad \text{or} \quad a = \dot{v} = \ddot{s}$$

Smarks

Unit 9: Kinematics 2 (variable acceleration) (Mechanics)

- 9a. Variable force; Calculus to determine rates of change for kinematics
- 9b: Use of integration for kinematics problems i.e. r = ∫ vdt, v = ∫ adt

Key Vocabulary

Distance, displacement, velocity, speed, constant acceleration, variable acceleration, retardation, deceleration, gradient, area, differentiate, integrate, rate of change, straight-line motion, with respect to time, constant of integration, initial conditions.

Variable acceleration 1

You can write displacement, velocity and acceleration as **functions of time**. This allows you to model the motion of an object which is moving in a straight line with variable acceleration.

Worked example

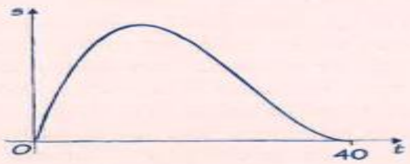
A pipe-inspection robot travels along a straight section of sewer pipe. At time t minutes the distance, s metres, of the robot from its starting position is given by

$$s = \frac{t^3 - 80t^2 + 1600t}{50}, \quad 0 \leqslant t \leqslant 40$$

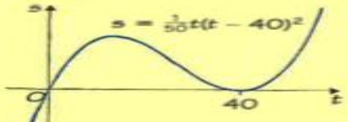
(a) Sketch a distance–time graph for the motion of the robot. **(3 marks)**

$$s = \frac{1}{50}t(t^2 - 80t + 1600)$$

$$= \frac{1}{50}t(t - 40)^2$$



You need to know more information about how the function behaves before you can sketch the distance–time graph. You can take out a factor of t and then factorise the quadratic factor. The graph of $s = \frac{1}{50}t(t - 40)^2$ is a cubic graph with a positive coefficient of t^3. It crosses the t-axis at $t = 0$ and touches it at $t = 40$:



There is more about sketching graphs of cubic functions on page 12.

The model provided is only valid for $0 \leqslant t \leqslant 40$. Make sure you only sketch the section of the graph for these values of t.

(b) Find the maximum distance of the robot from its starting position. **(6 marks)**

$$\frac{ds}{dt} = \frac{1}{50}(3t^2 - 160t + 1600)$$

$$0 = \frac{1}{50}(3t - 40)(t - 40)$$

$$t = \frac{40}{3} \text{ or } \cancel{t = 40}$$

When $t = \frac{40}{3}$,

$$s = \frac{1}{50}\left(\left(\frac{40}{3}\right)^3 - 80\left(\frac{40}{3}\right)^2 + 1600\left(\frac{40}{3}\right)\right)$$

$$= 189.62\ldots$$

The maximum distance of the robot from its starting point is 190 m (3 s.f.)

From your sketch you can see that $t = 40$ represents a minimum value, so reject that solution.

Maxima and minima

You can use differentiation to find any maximum and minimum values of a function of time. To find the maximum value of $s = f(t)$, differentiate and set $f'(t) = 0$. Solve to find t and then substitute this value back into $f(t)$.

The derivative of the displacement function represents the **velocity**. There is more about this on the next page. There is more about differentiating and finding minima and maxima on pages 39 and 40.

Completing the function

An indefinite integral always produces a function with a **constant of integration**. You need to find this constant in order to use the function to solve numerical problems. When you integrate to find velocity or displacement, you might need to use a given value to find the constant of integration. If this is the value when $t = 0$ it is called an **initial condition**.

There is more about finding functions by integrating on page 43.

Variable acceleration 2

Velocity is the rate of change of displacement, and acceleration is the rate of change of velocity.

$$v = \frac{ds}{dt} \text{ and } a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Integration is the **inverse** of differentiation, so you can integrate an expression for the velocity to find the displacement, or you can integrate an expression for the acceleration to find the velocity.