YEAR 9 – REASONING WITH GEOMETRY...
Deduction

What do I need to be able to do?

By the end of this unit you should be able to:

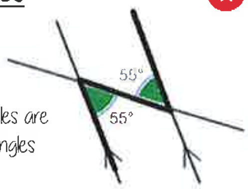
- Identify angles in parallel lines
- Solve angle problems
- Make conjectures with angles
- Make conjectures with shapes

Keywords

Parallel: two straight lines that never meet with the same gradient
Perpendicular: two straight lines that meet at 90°
Transversal: a line that crosses at least two other lines
Sum: the result of adding two or more numbers
Conjecture: a statement that might be true but is not proven
Equation: a statement that says two things are equal
Polygon: a 2D shape made from straight edges
Counterexample: an example that disproves a statement

Alternate angles

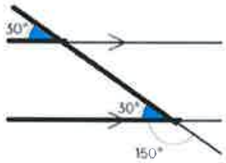
R



Because alternate angles are equal the highlighted angles are the same size

Corresponding angles

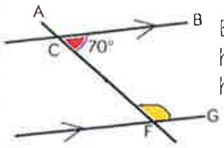
R



Because corresponding angles are equal the highlighted angles are the same size

Co-interior angles

R



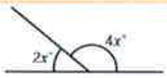
Because co-interior angles have a sum of 180° the highlighted angle is 110°

As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Solving angle problems

Angles on a straight Line
180°

Link angle facts to algebra



Form an equation

$$2x + 4x = 180°$$

State the reason

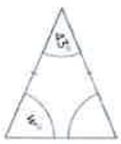
The sum of angles on a straight line is 180°

Solve

$$2x + 4x = 180°$$
$$6x = 180°$$
$$x = 30°$$

Vertically opposite angles
Equal

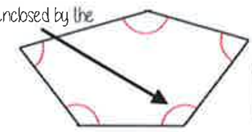
Angles around a point
360°



Triangles
Sum of angles is 180°

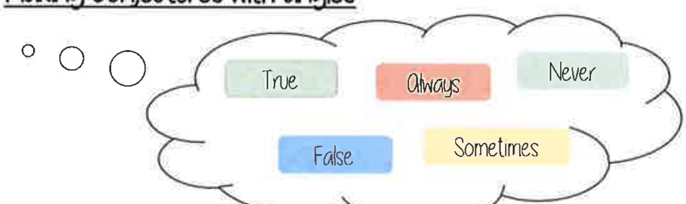
Isosceles have the same base angles

Interior Angles
The angles enclosed by the polygon



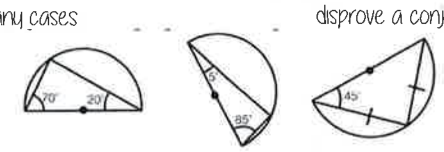
(number of sides – 2) x 180

Making conjectures with angles



Proving a conjecture
A pattern is noticed for many cases

Disproving a conjecture
Only one counterexample is needed to disprove a conjecture



Apply the angle rules → Test the theory → Make conjecture

The sum of angles in a triangle is 180°

$180 - 70 - 20 = 90$
$180 - 85 - 5 = 90$
$180 - 45 - 45 = 90$

The angle that meets the circumference in a semi circle is 90°

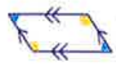
Making conjectures with shapes

Keywords and facts to recall with shape

Area: the amount of space inside a shape
Perimeter: the length around a shape
Regular Polygons: All sides and angles are equal

Quadrilateral Facts

Square
All sides equal size
All angles 90°
Opposite sides are parallel



Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Rectangle
All angles 90°
Opposite sides are parallel

Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Rhombus
All sides equal size
Opposite angles are equal

YEAR 9 — REASONING WITH GEOMETRY...

Rotation & Translation

What do I need to be able to do?

By the end of this unit you should be able to:

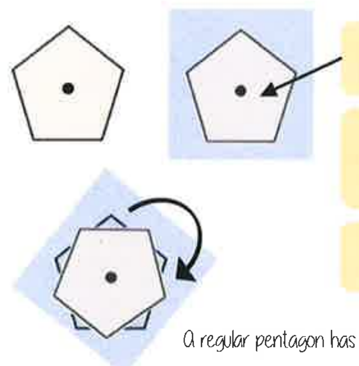
- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement
Symmetry: when two or more parts are identical after a transformation
Regular: a regular shape has angles and sides of equal lengths
Invariant: a point that does not move after a transformation
Vertex: a point two edges meet
Horizontal: from side to side
Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



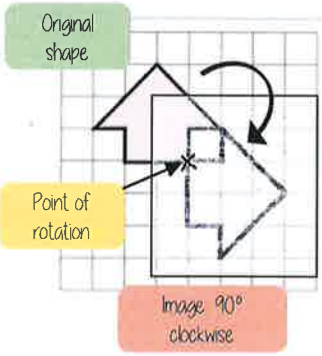
1 Trace your shape (mark the centre point)

2 Rotate your tracing paper on top of the original through 360°

3 Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

Rotate from a point (in a shape)

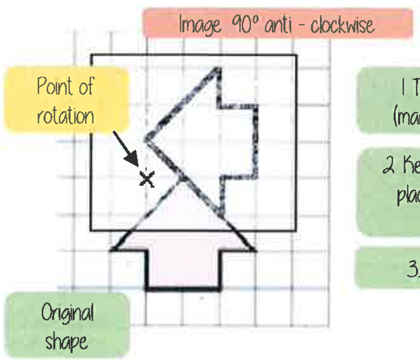


1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape

Rotate from a point (outside a shape)



1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

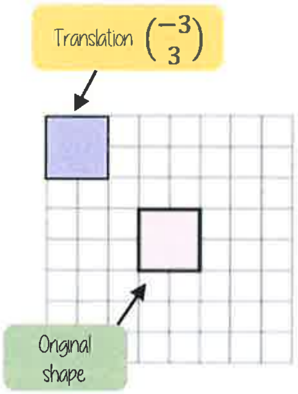
3 Draw the new shape

Translation and vector notation

Vector Notation → $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

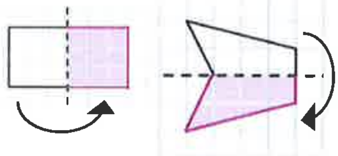
How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)



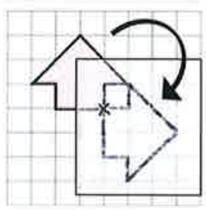
Every vertex has been translated by the same amount

Compare rotations and reflections



Reflections are a mirror image of the original shape

Information needed to perform a reflection
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation
- Point of rotation
- Direction of rotation
- Degrees of rotation

YEAR 9 — REASONING WITH GEOMETRY…

Pythagoras' theorem

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

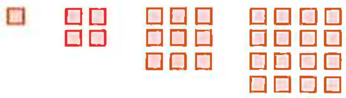
Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots R



This can also be written as 6^2

1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10
1	4	9	16	25	36	49	64	81	100

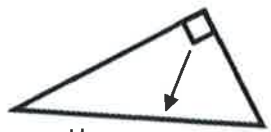
Square numbers

$\sqrt{\ }$ is the square root symbol

eg $\sqrt{64} = 8$

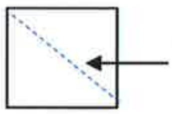
Because 8 × 8 = 64

Identify the hypotenuse



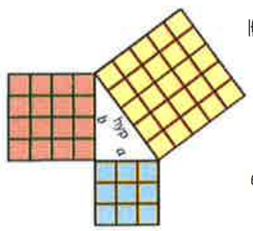
Hypotenuse

The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



a = 3 b = 4 c = 5

If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

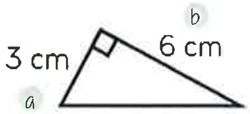
eg $a^2+b^2 = hypotenuse^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$3^2+6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

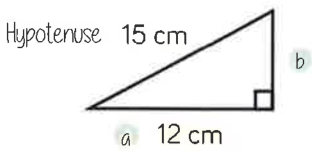
$45 = \text{hypotenuse}^2$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

$\sqrt{45} = \text{hypotenuse}$

$6.71cm = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$12^2+b^2 = 15^2$

1 Substitute in the values you are given

$144 + b^2 = 225$

$-144 \qquad -144$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

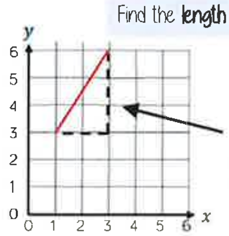
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram

The line segment is the **hypotenuse**

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes