YEAR 10 – PROPORTION...

Ratios and fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare quantities using ratio
- Link ratios and fractions and make comparisons
- Share in a given ratio
- Link Ratio and scales and graphs
- Solve problems with currency conversions
- Solve 'best buy' problems
- Combine ratios

Keywords

Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios
Integer: whole number, can be positive, negative or zero
Fraction: represents how many parts of a whole
Denominator: the number below the line on a fraction The number represent the total number of parts
Numerator: the number above the line on a fraction The top number Represents how many parts are taken
Origin: (0,0) on a graph The point the two axes cross
Gradient: The steepness of a line

Compare with ratio (R)

"For every dog there are 2 cats"

Dogs: Cats

1:2

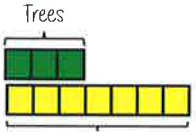
The ratio has to be written in the same order as the information is given
e.g. 2:1 would represent 2 dogs for every 1 cat

Units have the be of the same value to compare ratios

Ratios and fraction (R)

Trees: Flowers

3 : 7



Ratio

Fraction of trees

Number of parts of in group / Total number of parts

3/10

Fraction

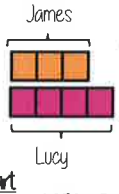
Sharing a whole into a given ratio (R)

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



£350

Find the value of one part

Whole £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part
= £50

Put back into the question

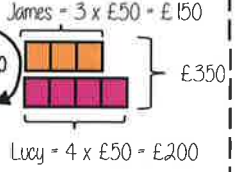
James: Lucy

x 50 3 : 4 x 50

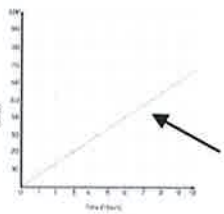
£150:£200

James = 3 x £50 = £150

Lucy = 4 x £50 = £200



Ratio and graphs (R)



Graphs with a constant ratio are directly proportional
- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale (R)

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image	Real life
1cm	30cm
10cm	300cm

(÷10 / ×10)

Conversion between currencies (R)



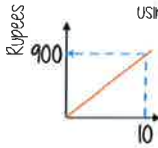
For every £1 I have 90 Rupees

£1 = 90 Rupees ← Currency is directly proportional

£1 = 90 Rupees (×10 / ×10)

£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

£1 = 90 Rupees (×7 / ×7)

630 ÷ 90 = 7

£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that **this part has to be 1 unit** Therefore **Divide by 4**

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in **proportion**

the n part does not have to be an integer for this type of question

Best buys


	4 pens costs £2.60	10 pens costs £6.00
"1 pen costs..."	£2.60 ÷ 4 = £0.65	£6.00 ÷ 10 = £0.60
"1 pound buys..."	4 ÷ 2.60 = 1.54 pens	10 ÷ 6 = 1.67 pens

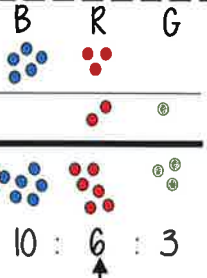
You could work out how much 40 pens are and then compare

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

The best value means £1 buys you more pens

Combining ratios



The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green

10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

YEAR 10 – PROPORTION...

Percentages and Interest

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

Keywords

Exponent: how many times we use a number in multiplication It is written as a power
Compound interest: calculating interest on both the amount plus previous interest
Depreciation: a decrease in the value of something over time
Growth: where a value increases in proportion to its current value such as doubling
Decay: the process of reducing an amount by a consistent percentage rate over time
Multiplier: the number you are multiplying by
Equivalent: of equal value

Compare FDP (R)

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 - 100 → 70 out of 100 squares → 70 "hundredths" - 7 "tenths" 0.7

→ 70 hundredths - 70%

Using a calculator

Convert to a decimal

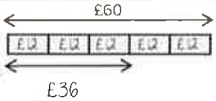
This will give the answer in the simplest form

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3} = 0.3333333$
$= 0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount (R)

Find $\frac{3}{5}$ of £60



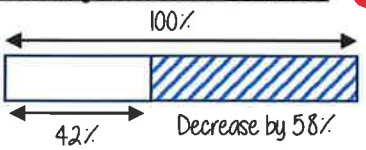
Remember $\frac{3}{5} = 60\% = 0.6$

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5} = 60\% = 0.6$

60% of £60
= 0.6 x 60
= £36

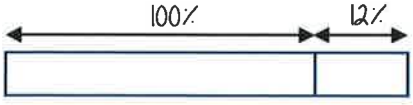
Percentage increase/decrease (R)



Decrease by 58%

100% - 58% = 42%
100 – 058 = 042

Multiplier Less than 1

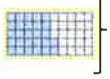


Increase by 12%

100% + 12% = 112%
100 + 012 = 112

Multiplier More than 1

Express as a percentage (R)



27 per every 50 shaded → $\frac{27}{50}$

54 per every 100 shaded → $\frac{54}{100}$

54%

$\frac{13}{30}$ →  → $\frac{13}{30}$ → × 100

Can't use equivalence easily to find 'per hundred'

43.3333...%
43%

Decimal percentages are still a percentage

Simple and compound interest

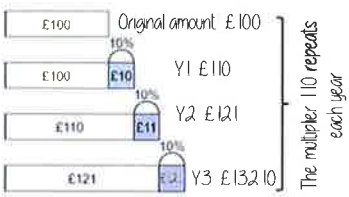
Simple Interest

James invests £2000 at 5% simple interest

The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years



Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 x 110 x 110 x 110

£100 x 110 ^3

Original amount — Repeated multiplier — Number of occurrences

Depreciation

Depreciation calculations use multipliers **less than 1**

Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

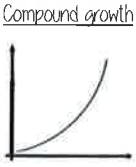
e.g Increase of 10% then a reduction of 10%

x 110 x 09

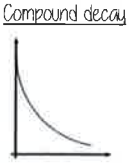
x 099 The multiplier

Growth and decay

Compound growth



Compound decay



Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

Find the original value

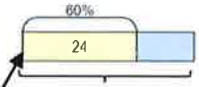
Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly Her score was 24. How many questions were on the test

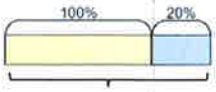
Original x 06 = 24

24 ÷ 06 = 40 marks
10% = 6
100% = 40



Total questions on test

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?



£3000

Original x 1.2 = 3000

120% = £3000
10% = £250
100% = £2500

YEAR 10 — PROPORTION...

Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Add, Subtract and multiply fractions
- Find probabilities using likely outcomes
- Use probability that sums to 1
- Estimate probabilities
- Use Venn diagrams and frequency trees
- Use sample space diagrams
- Calculate probability for independent events
- Use tree diagrams

Keywords

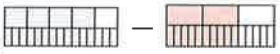
Event: one or more outcomes from an experiment
Outcome: the result of an experiment
Intersection: elements (parts) that are common to both sets
Union: the combination of elements in two sets
Expected Value: the value/ outcome that a prediction would suggest you will get
Universal Set: the set that has all the elements
Systematic: ordering values or outcomes with a strategy and sequence
Product: the answer when two or more values are multiplied together

Add, Subtract and multiply fractions

R

Addition and Subtraction

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

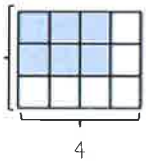
Use equivalent fractions to find a common multiple for both denominators

Multiplication

$\frac{3}{4} \times \frac{2}{3}$

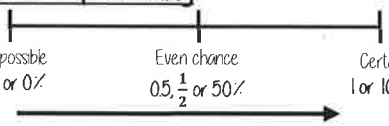
$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$ ← Parts shaded; ← Total number of parts in the diagram

Modelled:



Likeliness of a probability

R



The more likely an event the further up the probability it will be in comparison to another event (it will have a probability closer to 1)

Sum to 1

R

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability — What we expect to happen

Experimental probability — What actually happens when we try it out

The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
Theoretical probability is proportional

Sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

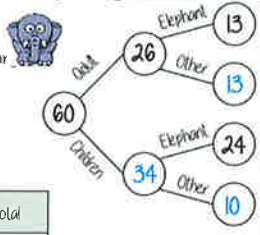
P (Even number and tails) = $\frac{3}{12}$

R

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Two-way table

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

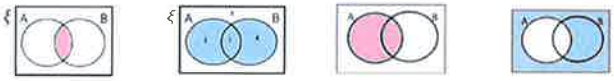
Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

P(Adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Venn diagram



in set A AND set B	in set A OR set B	in set A	NOT in set A
$P(A \cap B)$	$P(A \cup B)$	$P(A)$	$P(A')$

Independent events

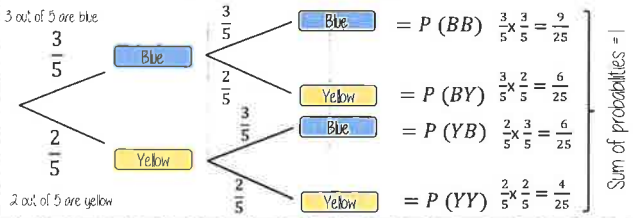
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$P(A \text{ and } B) = P(A) \times P(B)$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability

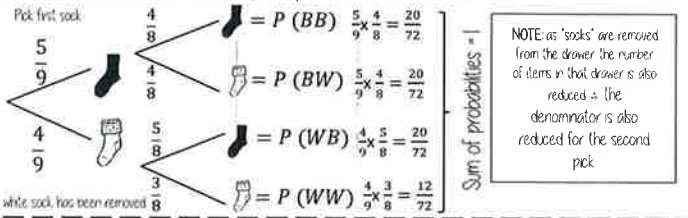


Dependent events

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks, Jamie picks 2 socks from the drawer



NOTE: as 'socks' are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick