YEAR 11 — DEVELOPING ALGEBRA...

Simultaneous Equations

What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position
Intersection: the point two lines cross or meet

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

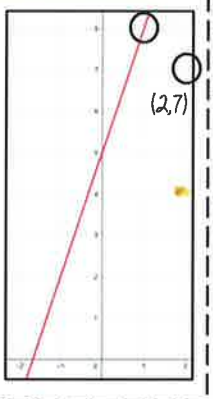
Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5



Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5

Substituting known variables

Stephanie knows the point x = 4 lies on that line. Find the value for y

A line has the equation $3x + y = 14$

$x = 4$

$3x + y = 14$

$3(4) + y = 14$

$12 + y = 14$

$-12 \quad -12$

$y = 2$

Two different variables, two solutions

Substituting in an expression

$x = 2y$

$x + y = 30$

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value

$x = 2y$

$x + y = 30$

$3y = 30$

$\div 3 \quad \div 3$

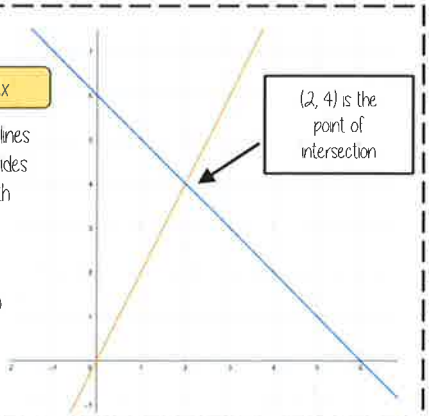
$y = 10$

$x = 20$

Solve graphically

$x + y = 6$ $y = 2x$

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations



The solution that satisfies both equations is

$x = 2$ and $y = 4$

Solve by subtraction

$3x + 2y = 18$
$-\ x + 2y = 10$

$2x = 8$

$\div 2 \quad \div 2$

$x = 4$

$x + 2y = 10$
$(4) + 2y = 10$

$-4 \quad -4$

$2y = 6$

$\div 2 \quad \div 2$

$y = 3$

$x = 4$
$y = 3$

Solve by addition

Addition makes zero pairs

$3x + 2y = 16$
$+\ 6x - 2y = 2$

$9x = 18$

$\div 9 \quad \div 9$

$x = 2$

$3x + 2y = 16$
$3(2) + 2(y) = 16$
$6 + 2y = 16$

$-6 \quad -6$

$2y = 10$

$y = 5$

$x = 2$
$y = 5$

Solve by adjusting one

$h + j = 12$
$2h + 2j = 29$

No equivalent values

$2h + 2j = 24$
$2h + 2j = 29$

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$
$5x - 2y = -7$

Use LCM to make equivalent x OR y values
Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$
$15x - 6y = -21$

Now solve by addition

Addition makes zero pairs

YEAR 11 — GEOMETRY...

Working with circles

What do I need to be able to do?

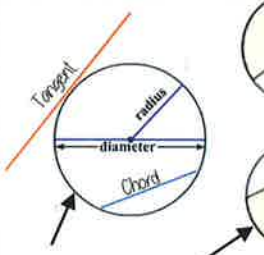
By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere
- Understand and use surface area of a cone, cylinder and sphere

Keywords

Circumference: the length around the outside of the circle – the perimeter
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
Frustrum: a pyramid or cone with the top cut off
Hemisphere: half a sphere
Surface area: the total area of the surface of a 3D shape

Parts of a circle (R)



Sector (part of the circle made from two radii)

Segment (part of the circle made from a chord)

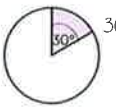
Circumference

An arc is a part of the circumference

Fractional parts of a circle

A circle is made up of 360°

Formula to remember
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$

$\frac{270}{360}$ of a full circle (in degrees)

$\frac{6}{8}$ of a full circle (in equal parts)

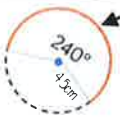
$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

θ represents the degrees in the sector

Arc length

Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$



Arc length = $\frac{\theta}{360} \times$ circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi = 6\pi$

Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360} \times$ circumference $+ 2r$ $= 6\pi + 9$

Sector area

Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$

Sector area = $\frac{\theta}{360} \times$ area of circle

$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = 12\pi$

Volume of a cone and a cylinder

Volume Cylinder = $\pi r^2 h$

A cylinder is a prism – cross section is a circle

Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base

$V = \pi r^2 h$
$= \pi \times 4^2 \times 10$
$= \pi \times 160$
$= 160\pi$ cm^3

Give your answer in terms of π' means NOT in terms of pi $= 502.7$cm^3

The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Volume of a sphere

Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value

Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

$= \frac{4}{3} \times \pi \times 27 = 36\pi$

A hemisphere is half the volume of the overall sphere $= 36\pi \div 2$

$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders

Surface area of a sphere

Surface area = $4\pi r^2$

Radius = 5cm

Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

$= 100\pi$

The curved surface area of a sphere

A hemisphere has the curved surface AND a flat circular face

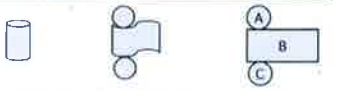
$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $= 75\pi$

Surface area of cones and cylinders

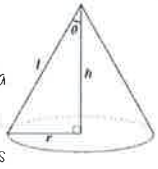
Surface area cylinder = $2\pi r^2 + \pi dh$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles

Curved surface area Cone = πrl



Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)