Year 9
Knowledge Organisers

Block: Spring 1
Reasoning with Number

- Numbers
- Using percentages
- Maths & money

St marks

ACADEMY

YEAR 9 — REASONING WITH NUMBER...
Numbers

What do I need to be able to do?

By the end of this unit you should be able to:
- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

Keywords

Integer: a whole number that is positive or negative
Rational: a number that can be made by dividing two integers
Irrational: a number that cannot be made by dividing two integers
Inverse operation: the operation that reverses the action
Quotient: the result of a division
Product: the result of a multiplication
Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number

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Integers, real and rational numbers

Rational – root word: ratio

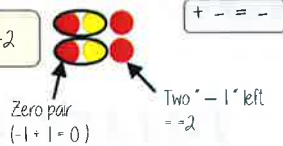Real numbers: $\frac{2}{3}$ stems from $2\frac{1}{3}$ ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat

The square root of a negative is not a real number and cannot be found

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HCF/LCM (R)

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

| 18 | 1, 2, 3, 6, 9, 18 |
| 30 | 1, 2, 3, 5, 6, 10, 15, 30 |

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

| 9 | 9, 18, 27, 36, 45, 54 |
| 12 | 12, 24, 36, 48, 60 |

LCM = 36

The first time their multiples match

Standard form (R)

Any number between 1 and less than 10

$A \times 10^n$ ← Any integer

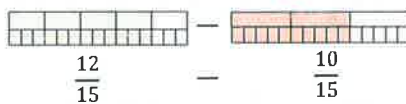$6 \times 10^5 + 8 \times 10^5$
= 600000 + 800000
= 1400000
= 1.4 × 10⁵

$(1.5 \times 10^5) \div (0.3 \times 10^3)$
15 ÷ 0.3 × 10⁵ ÷ 10³
= 5 × 10²

Directed number (R)

Addition

$2 + -4 = -2$

Generalisation: $+ - = -$

Zero pair $(-1 + 1 = 0)$

Two "−1" left = −2

Subtraction

⊖ = 2

Generalisation: $- - - = +$

Representation for calculation

$2 - -1 = 3$

Take away one ●

Start with the representation of 2

"Subtract" – means take away or remove

⊖ = 3

Multiplication

$-2 \times -3 = 6$
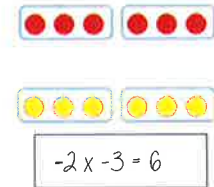
The act of making counters into their negative is turning them over

Divisions are the inverse operations

| a = 5 | b = -4 |

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$

Addition/ Subtraction of fractions (R)

$\frac{4}{5} - \frac{2}{3}$

$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

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Multiplication/ Division of fractions (R)

Shade in 3 parts

Repeat it on this many rows
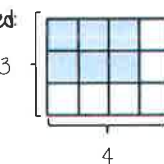
$\frac{3}{4} \times \frac{2}{3}$

This many columns

This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Modelled:

Total number of parts in the diagram
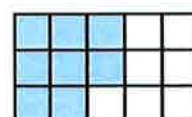
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

Multiplying by a reciprocal gives the same outcome

$\frac{2}{5} \times \frac{4}{3}$

Represented

$= \frac{8}{15}$

YEAR 9 — REASONING WITH NUMBER...
Using Percentages

What do I need to be able to do?

By the end of this unit you should be able to:
- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

Percent: parts per 100 – written using the % symbol

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals

Fraction: a fraction represents how many parts of a whole value you have

Equivalent: of equal value

Reduce: to make smaller in value

Growth: to increase/ to grow

Integer: whole number, can be positive, negative or zero

Invest: use money with the goal of it increasing in value over time (usually in a bank)

Multiplier: the number you are multiplying by

Profit: the income take away any expenses/ costs

FDP Equivalence ®

One Whole = 1

Percentage
100% = a whole = 100 hundredths

10 hundredths
10 out of 100
10%

$\frac{10}{100} = \frac{1}{10} = 0.10$

One **hundredth**
(one whole split into 100 equal parts)

ones	tenths	hundredths
•	•	

Converting FDP ®

$\frac{70}{100}$ → This also means → 70 out of 100 squares 70 "hundredths" = 7 "tenths" 0.7 → 70 hundredths = 70%

Using a calculator

S⇌D Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3} = 0.3333333$
$\frac{1}{3} = 0.\dot{3}$
The dot above the 3

Percentage Increase/ Decrease ®

Decrease
100%

Decrease by 58%

42%

$100 - 0.58 = 0.42$ ← **Multiplier** Less than 1

Increase
100%

Increase by 12%

$100\% + 12\% = 112\%$
$100 + 0.12 = 112$ ← **Multiplier** More than 1

Percentage change ®

I bought a phone for £200. A year later sold it for £125

100%
£200
£125

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

All values of change compare to the ORIGINAL value

Difference in values / **Original value** × 100

I bought a house for £180,000, later sold it for £216,000

100%
£180,000

Percentage profit

Money made (profit value) → $\frac{36000}{180000} \times 100 = 20\%$

Reverse Percentages

40% of my number is 16. What am I thinking of?

Original Number (100%)

| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |

16

40% = 16
10% = 4
100% = 40

Try to scale down to 10% or 1% and then scale back up to 100%

140% of my number is 84. What is the original number?

Original Number (100%)

| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |

84

140% = 84
10% = 6
100% = 60

YEAR 9 — REASONING WITH NUMBER...
Maths & Money

What do I need to be able to do?

By the end of this unit you should be able to:
- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

Keywords

Credit: money being placed into a bank account
Debit: money that leaves a bank account
Balance: the amount of money in a bank account
Expense: a cost/ outgoing
Deposit: an initial payment (often a way of securing an item you will later pay for)
Multiplier: a number you are multiplying by (Multiplier more than 1 = increasing, less than 1 = decreasing)
Per Annum: each year
Currency: the type of money a country uses
Unitary: one — the cost of one

Bills and Bank Statements

Bills — tell you the amount items cost and can show how much money you need to pay

Some can include a total

Look for different units
(Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

| Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Setp	Bday Money	£15		£915

Simple Interest

For each year of investment the interest remains the same.

$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account
e.g Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned **£120** interest
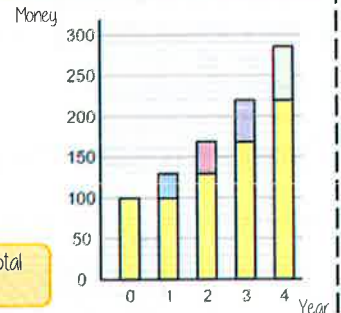At the end of year 4 they have **£220**
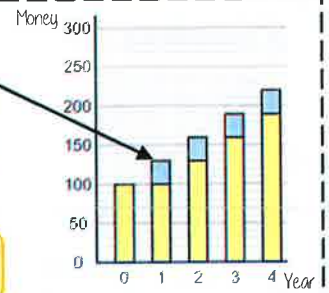
Compound Interest

Interest is added to the current value of investment at the end of each year so the next year's interest is greater

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = £285.61$$

This account has **£285.61** in total at the end of the 4 years

Value Added Tax (VAT)

VAT is payable to the government by a business In the UK VAT is 20% and added to items that are bought

Essential items such as food do not include VAT

Wages and Taxes

Salaries fall into tax brackets — which means they pay this much each month from their salary

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:
Time and a half — means 1.5 times their hourly rate
Double — 2 times their hourly rate

Exchange Rates

£1 — x 1.4 → $1.40
£1 — x 200 → £200
$1.40 — x 200 → $280
£200 — x 1.4 → £280
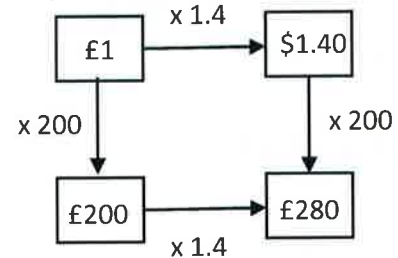
When making estimates it is also useful to use estimates to check if our solution is reasonable

Use inverse operations to reverse the exchange process

Common Currencies
United Kingdom	£	Pounds
United States of America	$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges	5 cupcakes
£1	£1.20

To calculate unit per cost you divide by the cost

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units

4 = £1.00 ⟩ ÷ 2
2 = £0.50
1 = £0.25 ⟩ ÷ 2

5 = £1.20 ⟩ ÷ 5
1 = £0.20

Cost per Unit