

Big Idea: Algebra

Key Vocabulary

Simplify, Substitute, Equivalent, Coefficient, Product, Highest Common Factor (HCF), Inequality.

What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

DOUBLE BRACKETS — you get 4 terms, and usually 2 of them combine to leave 3 terms.

There's a handy way to multiply out double brackets — it's called the **FOIL method** and works like this:

First — multiply the first term in each bracket together

Outside — multiply the outside terms (i.e. the first term in the first bracket by the second term in the second bracket)

Inside — multiply the inside terms (i.e. the second term in the first bracket by the first term in the second bracket)

Last — multiply the second term in each bracket together

Expand and simplify $(n-1)(2n+5)$

$$\begin{aligned} (n-1)(2n+5) &= (n \times 2n) + (n \times 5) + (-1 \times 2n) + (-1 \times 5) \\ &= 2n^2 + 5n - 2n - 5 \\ &= 2n^2 + 3n - 5 \end{aligned}$$

EXAMPLE:

The two n terms combine together.

Form expressions


For unknown variables, a letter is normally used in its place

More than — ADD
Less than/ difference — SUBTRACT

e.g 4 more than t $\longrightarrow t + 4$
8 less than k $\longrightarrow k - 8$

Only similar terms can be grouped together

e.g Find the perimeter of this shape
(Perimeter = length around outside of shape)



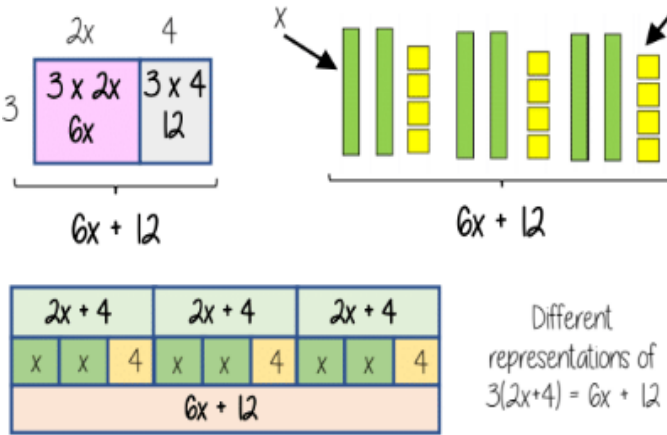
$t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$

Directed numbers

$++ \longrightarrow +$
 $-- \longrightarrow +$
 $+- \longrightarrow -$
 $-+ \longrightarrow -$

e.g $a = -5$ and $b = 2$
 $a^2 = a \times a = -5 \times -5 = 25$
 $b + a = 2 + -5 = -3$

Multiply single brackets



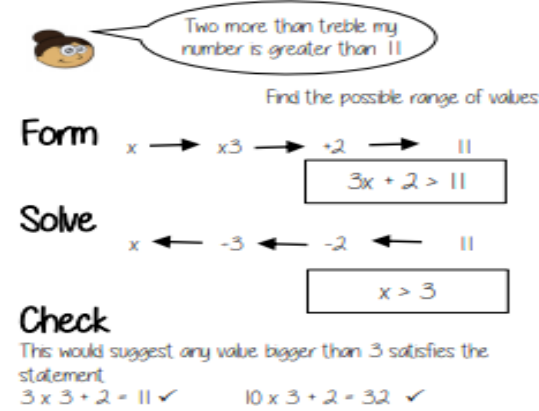
Simple Inequalities

$<$ less than \leq Less than or equal to
 $>$ More than \geq More than or equal to



Note: $x < 10$ and $10 > x$ represent the same values
 $x + 2 \leq 20$ "my value + 2 is less than or equal to 20"
 $x \leq 18$
The biggest the value can be is 18

Form and solve inequalities



Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

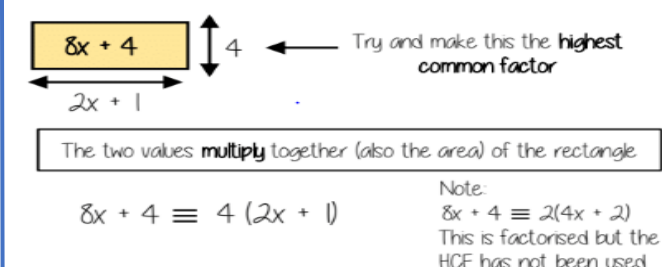
Equation
A statement that two things are equal

Term
A single number or variable

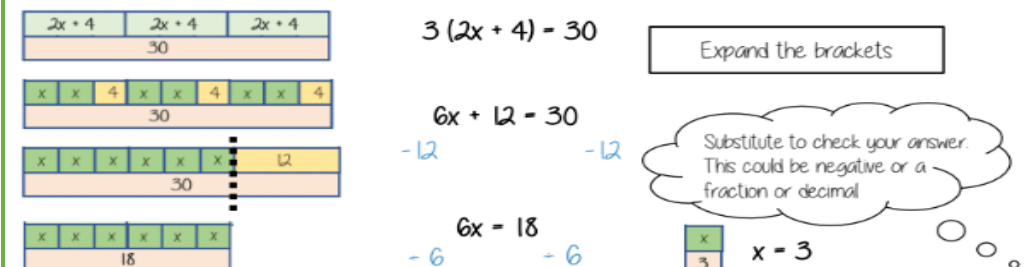
Identity
An equation where both sides have variables that cause the same answer includes \equiv

Formula
A rule written with all mathematical symbols e.g. area of a rectangle $A = b \times h$

Factorise into a single bracket



Solve equations with brackets



Key Vocabulary

Sequence, Term, Position, Linear, Non-linear, Difference, Arithmetic, Geometric, Base, Power, Exponent, Indices, Coefficient, Simplify, Product.

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence – look out for this type of sequence

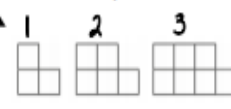
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms



Sequence in a table and graphically

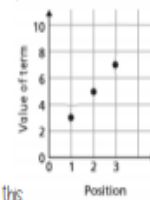
Position: the place in the sequence



Terms: the number or variable (the number of squares in each image)

Position	1	2	3
Term	3	5	7

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

H Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20, ...

4n

7, 11, 15, 19, 22

This has the same constant difference – but is 3 more than the original sequence

4n + 3

Sequences from algebraic rules

This is substitution!

3n + 7

This will be linear - note the single power of n. The values increase at a constant rate

2n - 5 →

Substitute the number of the term you are looking for in place of 'n'

- eg
- 1st term - 2(1) - 5 = -3
 - 2nd term - 2(2) - 5 = -1
 - 100th term - 2(100) - 5 = 195

3n² + 7

This is not linear as there is a power for n

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n - 4?

3n - 4 = 201

Term to check

Algebraic rule

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Complex algebraic rules

Misconceptions and comparisons

2n²

2 times whatever n squared is

(2n)²

2 times n then square the answer

- eg
- 1st term - 2 x 1² = 2
 - 2nd term - 2 x 2² = 8
 - 100th term - 2 x 100² = 2000

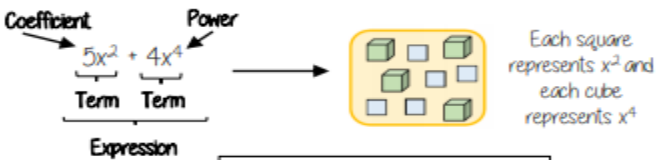
- eg
- 1st term - (2 x 1)² = 4
 - 2nd term - (2 x 2)² = 16
 - 100th term - (2 x 100)² = 40000

n(n + 5)

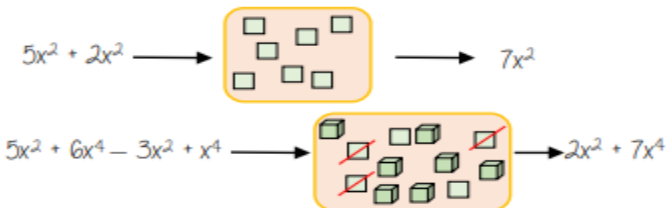
- eg
- 1st term - 1(1 + 5) = 6
 - 2nd term - 2(2 + 5) = 14
 - 100th term - 100(100 + 5) = 10500

You don't need to expand the expression

Addition/ Subtraction with indices



Only similar terms can be simplified. If they have different powers, they are unlike terms



Multiply expressions with indices

$$4b \times 3a \equiv 4 \times b \times 3 \times a \equiv 4 \times 3 \times b \times a \equiv 12ab$$

$$5t \times 9t \equiv 5 \times t \times 9 \times t \equiv 5 \times 9 \times t \times t \equiv 45t^2$$

$$2b^4 \times 3b^2 \equiv 2 \times b \times b \times b \times b \times 3 \times b \times b \equiv 2 \times 3 \times b \times b \times b \times b \times b \times b \equiv 6b^6$$

There are often misconceptions with this calculation but break down the powers

Divide expressions with indices

$$\frac{24}{36} \rightarrow \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{3}}{\cancel{2} \times \cancel{3} \times \cancel{2} \times \cancel{3}} \rightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b} \times \cancel{b}} \rightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23a^7y^2}{5db^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Addition/ Subtraction laws for indices

$$3^5 \times 3^2 \rightarrow 3^7$$

$$-(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$$

The base number is all the same so the terms can be simplified

$$a^m \times a^n = a^{m+n}$$

$$3^5 \div 3^2 \rightarrow 3^3$$

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$$

$$a^m \div a^n = a^{m-n}$$