Knowledge Organiser: Mathematics Year 11 Higher Spring Term 2

Suggested websites: Maths Genie, Save My Exam and Corbett Maths

Big idea: Probability and Statistics

Key skills:

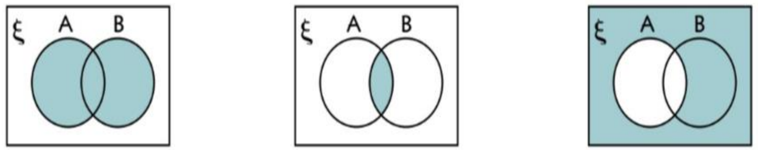
- Construction and Loci
- Construct and interpret tree diagrams
- Draw and interpret scatter graphs
- Draw and interpret Venn diagrams

Key Vocabulary

Loci, Construct, Complement, intersection, union, outcomes, probability, Venn diagram, sets

Show Sets on Venn Diagrams (Grade 5)

1) On a Venn diagram, each set is represented by a circle. The universal set is everything inside the rectangle.
2) The diagram can show either the actual elements of each set, or the number of elements in each set.



The union of sets A and B (written A ∪ B) contains all the elements in either set A or set B — it's everything inside the circles.

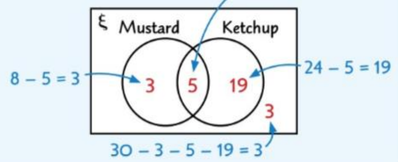
The intersection of sets A and B (written A ∩ B) contains all the elements in both set A and set B — it's where the circles overlap.

The complement of set A (written A') contains all members of the universal set that aren't in set A — it's everything outside circle A.

EXAMPLE: In a class of 30 pupils, 8 of them like mustard, 24 of them like ketchup and 5 of them like both mustard and ketchup.

a) Complete the Venn diagram below showing this information.

Start by filling in the overlap.



b) How many pupils like mustard or ketchup?

This is the number of pupils in the union of the two sets. 3 + 5 + 19 = 27

c) What is the probability that a randomly selected pupil will like mustard and ketchup?

5 out of 30 pupils are in the intersection. $\frac{5}{30} = \frac{1}{6}$

This is P(M ∩ K).

Tree Diagrams

Tree diagrams can really help you work out probabilities when you have a combination of events.

Remember These Four Key Tree Diagram Facts (Grade 5)

1) For branches which meet at a point, the probabilities add up to 1.

$\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

$\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

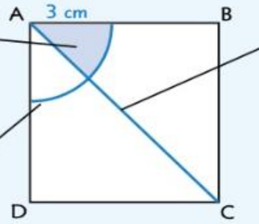
Total = 1

2) Multiply along the branches to get the end probabilities.
3) End probabilities add up to 1.
4) Pick the probability you need to answer the question.

Finding a Locus that Satisfies Lots of Rules (Grade 4)

In the exam, you might be given a situation with lots of different conditions, and asked to find the region that satisfies all the conditions. To do this, just draw each locus, then see which bit you want.

EXAMPLE: On the square below, shade the region that is within 3 cm of vertex A and closer to vertex B than vertex D.



The shaded area is the region you want.

Construct a quarter circle 3 cm from A using compasses — you want the region within it.

It's a square, so this diagonal is equidistant from B and D. The bit above the line is closer to B than D.

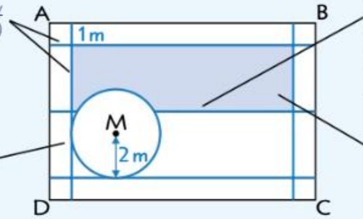
If it wasn't a square you'd have to CONSTRUCT the equidistant line with compasses using the method on p.89.

You might be given the information as a wordy problem — work out what you're being asked for and draw it.

EXAMPLE: Tessa is organising a village fete. The fete will take place on a rectangular field, shown in the diagram below. Tessa is deciding where an ice cream van can go. It has to be at least 1 m away from each edge of the field, and closer to side AB than side CD. There is a maypole at M, and the ice cream van must be at least 2 m away from the maypole. The diagram is drawn to a scale of 1 cm = 1 m. Show on it where the ice cream van can go.

Start by drawing lines 1 cm away from each side (to represent 1 m) — use a ruler to measure along each edge. The ice cream van must go within these lines.

Use compasses to draw a circle 2 cm away from M. The ice cream van has to go outside the circle.



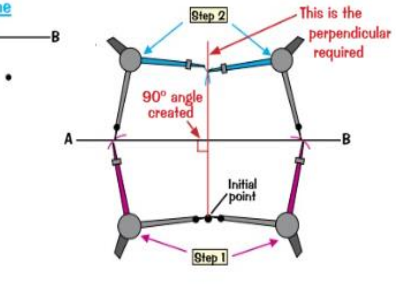
Draw a line equidistant from AB and CD (measure the length of side BC and divide it by two). The ice cream van has to go above this line.

The shaded area shows where the ice cream van can go.

In the examples above, the lines were all at right angles to each other, so you could just measure with a ruler rather than do constructions with compasses. If the question says "Leave your construction lines clearly visible", you'll definitely need to get your compasses out and use some of the methods on p.89-90.

Drawing the Perpendicular from a Point to a Line (Grade 4)

1) This is similar to the one above but not quite the same — make sure you can do both.
2) You'll be given a line and a point, like this:

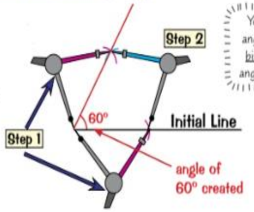


Constructing TWO PERPENDICULARS gives you PARALLEL LINES

If you draw another line perpendicular to the line you've just drawn, it'll be parallel to the initial line:

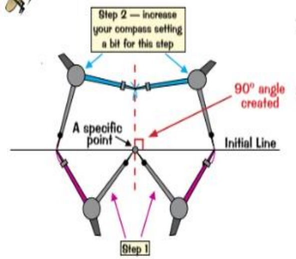
Constructing Accurate 60° Angles (Grade 4)

1) They may well ask you to draw an accurate 60° angle without a protractor.
2) Follow the method shown in this diagram (make sure you leave the compass settings the same for each step).



You can construct 30° angles and 45° angles by bisecting 60° and 90° angles (see previous page).

Constructing Accurate 90° Angles (Grade 4)



1) They might want you to construct an accurate 90° angle.
2) Make sure you can follow the method shown in this diagram.

The examiners WON'T accept any of these constructions done 'by eye' or with a protractor. You've got to do them the PROPER WAY, with compasses. DON'T rub out your compass marks, or the examiner won't know you used the proper method.

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Smarks

Big idea: Probability and Statistics

Key skills:
Construct and interpret a Histogram
Construct and interpret a box plot
Construct and interpret tree diagrams
Use the OR rule to solve probability questions
Draw and interpret Venn diagrams

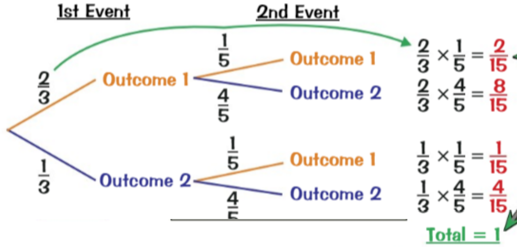
Key Vocabulary
Complement, intersection, union, outcomes, probability, Venn diagram, sets, Class width, frequency density, lower quartile, upper quartile, median, interquartile range, outliers

Tree Diagrams

Tree diagrams can really help you work out probabilities when you have a combination of events.

Remember These Four Key Tree Diagram Facts (Grade 7)

1) On any set of branches which meet at a point, the probabilities must add up to 1.

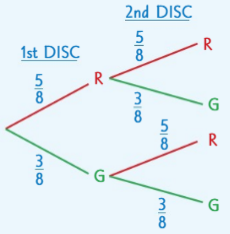


2) Multiply along the branches to get the end probabilities.

3) Check your diagram — the end probabilities must add up to 1.

4) To answer any question, add up the relevant end probabilities (see below).

EXAMPLE: A box contains 5 red discs and 3 green discs. One disc is taken at random and its colour noted before being replaced. A second disc is then taken. Find the probability that both discs are the same colour.



The probabilities for the 1st and 2nd discs are the same. This is because the 1st disc is replaced — so the events are independent.

P(both discs are red) = P(R and R) = $\frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

P(both discs are green) = P(G and G) = $\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

P(both discs are same colour) = P(R and R or G and G) $= \frac{25}{64} + \frac{9}{64} = \frac{34}{64} = \frac{17}{32}$

The AND Rule gives P(Both Events Happen) (Grade 6)

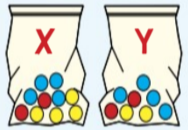
If two events, call them A and B, are independent then...

$$P(A \text{ and } B) = P(A) \times P(B)$$

If they're dependent, use the conditional probability rule (p.112).

The probability of events A AND B BOTH happening is equal to the two separate probabilities MULTIPLIED together.

EXAMPLE: Dave picks one ball at random from each of bags X and Y. Find the probability that he picks a yellow ball from both bags.



1) Write down the probabilities of the different events.
P(Dave picks a yellow ball from bag X) = $\frac{4}{10}$ = 0.4
P(Dave picks a yellow ball from bag Y) = $\frac{2}{8}$ = 0.25

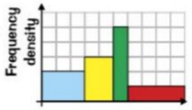
2) Use the formula. So P(Dave picks a yellow ball from both bags) = 0.4 × 0.25 = 0.1

Histograms and Frequency Density

A histogram is just a bar chart where the bars can be of different widths. This changes them from nice, easy-to-understand diagrams into seemingly incomprehensible monsters.

Histograms Show Frequency Density (Grade 7)

1) The vertical axis on a histogram is always called frequency density. You work it out using this formula:



Frequency Density = Frequency ÷ Class Width

Remember... 'frequency' is just another way of saying 'how much' or 'how many'.

2) You can rearrange it to work out how much a bar represents.

Frequency = Frequency Density × Class Width = AREA of bar

The OR Rule gives P(At Least One Event Happens) (Grade 6)

For two events, A and B...

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

The probability of EITHER event A OR event B happening is equal to the two separate probabilities ADDED together MINUS the probability of events A AND B BOTH happening.

If the events A and B can't happen together then P(A and B) = 0 and the OR rule becomes:

$$P(A \text{ or } B) = P(A) + P(B)$$

When events can't happen together they're called mutually exclusive.

EXAMPLE: A spinner with red, blue, green and yellow sections is spun — the probability of it landing on each colour is shown in the table. Find the probability of spinning either red or green.

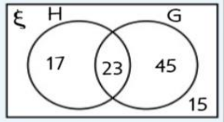
Colour	red	blue	yellow	green
Probability	0.25	0.3	0.35	0.1

The spinner can't land on both red and green so use the simpler OR rule. Just put in the probabilities.

P(red or green) = P(red) + P(green) = 0.25 + 0.1 = 0.35

Finding Probabilities from Venn Diagrams (Grade 8)

EXAMPLE: The Venn diagram on the right shows the number of Year 10 pupils going on the History (H) and Geography (G) school trips.



Find the probability that a randomly selected Year 10 pupil is:

a) not going on the History trip.
n(Year 10 pupils) = 17 + 23 + 45 + 15 = 100
n(Not going on History trip) = 45 + 15 = 60
P(Not going on History trip) = $\frac{60}{100} = \frac{3}{5}$ = 0.6

Use the formula from p.106 to find the probabilities.

b) not going on the History trip but going on the Geography trip.
n(Not going on History trip but going on Geography trip) = 45
P(Not going on History trip but going on Geography trip) = $\frac{45}{100} = \frac{9}{20}$ = 0.45

c) going on the Geography trip given that they're not going on the History trip.

Careful here — think of this as selecting a pupil going on the Geography trip from those not going on the History trip.

P(Going on Geography trip given not going on History trip) = $\frac{45}{45 + 15} = \frac{45}{60} = \frac{3}{4}$ = 0.75

You could also use the conditional probability formula and your answers to parts a) and b).

Box Plots

The humble box plot might not look very fancy, but it gives you a useful summary of a data set.

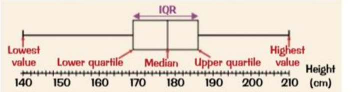
Box Plots show the Spread of a Data Set (Grade 6)

1) The lower quartile Q_1, the median Q_2 and the upper quartile Q_3 are the values 25% (¼), 50% (½) and 75% (¾) of the way through an ordered set of data. So if a set of data has n values, you can work out the positions of the quartiles using these formulas:

Q_1: (n + 1)/4 Q_2: (n + 1)/2 Q_3: 3(n + 1)/4

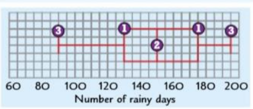
2) The INTERQUARTILE RANGE (IQR) is the difference between the upper quartile and the lower quartile and contains the middle 50% of values.

3) A box plot shows the minimum and maximum values in a data set and the values of the quartiles. But it doesn't tell you the individual data values.



EXAMPLE: This table gives information about the numbers of rainy days last year in some cities. On the grid below, draw a box plot to show the information.

Minimum number	90
Maximum number	195
Lower quartile	130
Median	150
Upper quartile	175



1. Mark on the quartiles and draw the box.
2. Draw a line at the median.
3. Mark on the minimum and maximum points and join them to the box with horizontal lines.