# Knowledge Organiser: Mathematics
# Year 11 Higher Autumn Term 1

**Suggested websites: Maths Genie, Save My Exam and Corbett Maths**

Sharks

## Big idea: Algebra

**Key skills:**

- Gradients and lines
- Non-linear Graphs
- Using Graphs

### Key Vocabulary

Linear, Gradient, Coordinate, Quadratic, Non Linear, Slope, Steepness, Reciprocal, Plott, Origin

## The 'Table of 3 Values' Method (Grade 3)

EXAMPLE: Draw the graph of $y = -2x + 4$ for values of $x$ from –1 to 4.

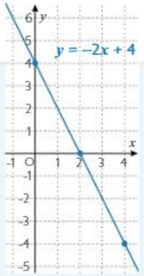
1) Draw up a table with three suitable values of x.

| $x$ | 0 | 2 | 4 |
|---|---|---|---|
| $y$ | | | |

2) Find the y-values by putting each x-value into the equation:
   When $x = 4$, $y = -2x + 4 = (-2 \times 4) + 4 = -4$

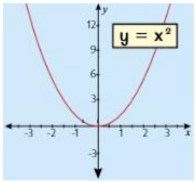
| $x$ | 0 | 2 | 4 |
|---|---|---|---|
| $y$ | 4 | 0 | –4 |

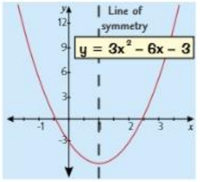
3) Plot the points and draw the line.
   The table gives the points (0, 4), (2, 0) and (4, –4)

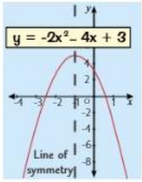



## Quadratic Graphs

Quadratic functions take the form $y$ = anything with $x^2$ (but no higher powers of x).
$x^2$ graphs all have the same symmetrical bucket shape.



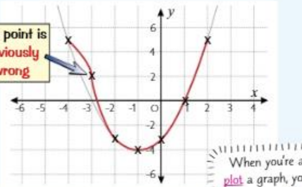

If the $x^2$ bit has a '–' in front of it then the bucket is upside down.

## Plotting Quadratics (Grade 5)

EXAMPLE: Complete the table of values for the equation $y = x^2 + 2x - 3$ and then plot the graph.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $y$ | 5 | 0 | -3 | -4 | -3 | 0 | 5 |

1) Substitute each x-value into the equation to get each y-value.
   E.g. $y = (-4)^2 + (2 \times -4) - 3 = 5$
2) Plot the points and join them with a completely smooth curve.

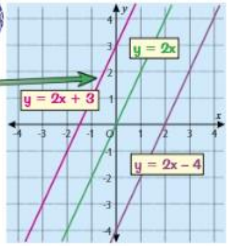



NEVER EVER let one point drag your graph off in some ridiculous direction. When a graph is generated from an equation, you never get spikes or lumps.

When you're asked to plot a graph, you should always draw it accurately using this method.

## Parallel Lines Have the Same Gradient (Grade 5)

Parallel lines all have the same gradient, which means their $y = mx + c$ equations all have the same value of $m$.
So the lines: $y = 2x + 3$, $y = 2x$ and $y = 2x - 4$ are all parallel.




EXAMPLE: Line J has a gradient of –0.25. Find the equation of Line K, which is parallel to Line J and passes through point (2, 3).

Lines J and K are parallel so their gradients are the same $\Rightarrow m = -0.25$

$y = -0.25x + c$

when $x = 2$, $y = 3$:
$3 = (-0.25 \times 2) + c \Rightarrow 3 = -0.5 + c$
$c = 3.5$

$y = -0.25x + 3.5$

1) First find the 'm' value for Line K.
2) Substitute the value for 'm' into $y = mx + c$ to give you the 'equation so far'.
3) Substitute the x and y values for the given point on Line K and solve for 'c'.
4) Write out the full equation.

## Find the Mid-Point Using The Average of the End Points (Grade 5)

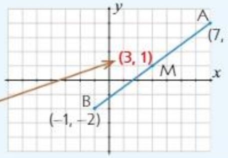
To find the mid-point of a line segment, just add the x-coordinates and divide by two, then do the same for the y-coordinates.

EXAMPLE: Points A and B are given by the coordinates (7, 4) and (–1, –2) respectively. M is the mid-point of the line segment AB. Find the coordinates of M.

Add the $x$-coordinate of A to the $x$-coordinate of B and divide by two to find the $x$-coordinate of the midpoint.

Do the same with the $y$-coordinates. $\left(\frac{7+-1}{2}, \frac{4+-2}{2}\right) = \left(\frac{6}{2}, \frac{2}{2}\right) = (3, 1)$

So the mid-point of AB has coordinates (3, 1)




## Use Ratios to Find Coordinates (Grade 7)

Ratios can be used to express where a point is on a line. You can use a ratio to find the coordinates of a point.

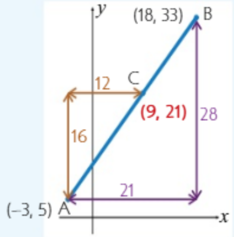
EXAMPLE: Point A has coordinates (–3, 5) and point B has coordinates (18, 33). Point C lies on the line segment AB, so that AC : CB = 4 : 3. Find the coordinates of C.

First find the difference between the coordinates of A and B:
Difference in $x$-coordinates: 18 – –3 = 21
Difference in $y$-coordinates: 33 – 5 = 28

Now look at the ratio you've been given: AC : CB = 4 : 3

The ratio tells you C is $\frac{4}{7}$ of the way from A to B — so find $\frac{4}{7}$ of each difference.
$x$: $\frac{4}{7} \times 21 = 12$
$y$: $\frac{4}{7} \times 28 = 16$

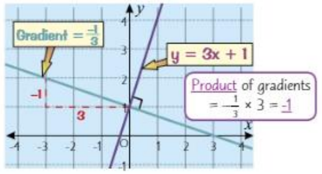
Now add these to the coordinates of A to find C.
$x$-coordinate: –3 + 12 = 9
$y$-coordinate: 5 + 16 = 21
Coordinates of C are (9, 21)




## Perpendicular Line Gradients (Grade 7)

Perpendicular lines cross at a right angle, and if you multiply their gradients together you'll get –1. Pretty nifty that.

**If the gradient of the first line is m, the gradient of the other line will be $-\frac{1}{m}$, because $m \times -\frac{1}{m} = -1$.**



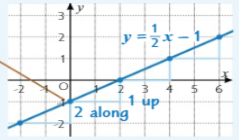

EXAMPLE: Lines A and B are perpendicular and intersect at (3, 3). If Line A has the equation $3y - x = 6$, what is the equation of Line B?

| | |
|---|---|
| Find 'm' (the gradient) for Line A. | $3y - x = 6 \Rightarrow 3y = x + 6 \Rightarrow y = \frac{1}{3}x + 2$, so $m_A = \frac{1}{3}$ |
| Find the 'm' value for the perpendicular line (Line B). | $m_B = -\frac{1}{m_A} = -1 \div \frac{1}{3} = -3$ |
| Put this into $y = mx + c$ to give the 'equation so far'. | $y = -3x + c$ |
| Put in the x and y values of the point and solve for 'c'. | $x = 3, y = 3$ gives: $3 = (-3 \times 3) + c \Rightarrow 3 = -9 + c \Rightarrow c = 12$ |
| Write out the full equation. | $y = -3x + 12$ |

## Using $y = mx + c$ (Grade 4)

EXAMPLE: Draw the graph of $4y - 2x = -4$.

1. Get the equation into the form $y = mx + c$. — $4y - 2x = -4 \rightarrow y = \frac{1}{2}x - 1$
2. Put a dot on the y-axis at the value of c. — 'c' = –1
3. Using m, go across and up or down a certain number of units. Make another dot, then repeat this step a few times in both directions. — Go 2 along and 1 up because 'm' = $+\frac{1}{2}$.
4. When you have 4 or 5 dots, draw a straight line through them.
5. Finally check that the gradient looks right. — A gradient of $+\frac{1}{2}$ should be quite gentle and uphill left to right — which it is, so it looks OK.




## Sketching Quadratics (Grade 7)

If you're asked to sketch a graph, you won't have to use graph paper or be dead accurate — just find and label the important points and make sure the graph is roughly in the correct position on the axes.

EXAMPLE: Sketch the graph of $y = -x^2 - 2x + 8$, labelling the turning point and $x$-intercepts with their coordinates.

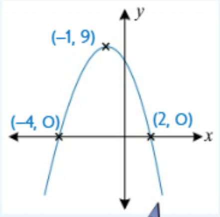
1. Find all the information you're asked for.

Solve $-x^2 - 2x + 8 = 0$ to find the $x$-intercepts (see p.34).
$-x^2 - 2x + 8 = -(x + 4)(x - 2) = 0$ so $x = -4$, $x = 2$

Use symmetry to find the turning point of the curve:
The $x$-coordinate of the turning point is halfway between -4 and 2. $x = \frac{-4+2}{2} = -1$
$y = -(-1)^2 - 2(-1) + 8 = 9$
So the turning point is (–1, 9).

2. Use the information you know to sketch the curve and label the important points.

The $x^2$ is negative, so the curve is n-shaped.



# Knowledge Organiser: Mathematics Year 11 Higher Autumn Term 1

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## Big idea: Algebra

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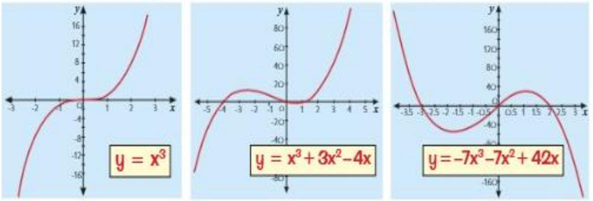
### Key Vocabulary

Circle, Reciprocal, Sine, Cosine, Tangent, Cubic, Cycle, Wave, Shift

## $x^3$ Graphs: $y = ax^3 + bx^2 + cx + d$ (b, c and d can be zero) (Grade 5)

All $x^3$ graphs (also known as cubic graphs) have a wiggle in the middle — sometimes it's a flat wiggle, sometimes it's more pronounced. $-x^3$ graphs always go down from top left, $+x^3$ ones go up from bottom left.

Note that $x^3$ must be the highest power and there must be no other bits like $1/x$ etc.

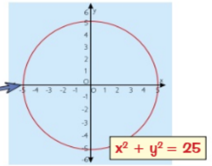



## Circles: $x^2 + y^2 = r^2$ (Grade 7)

**The equation for a circle with centre (0, 0) and radius r is: $x^2 + y^2 = r^2$**

$x^2 + y^2 = 25$ is a circle with centre (0, 0). $r^2 = 25$, so the radius, r, is 5.

$x^2 + y^2 = 100$ is a circle with centre (0, 0). $r^2 = 100$, so the radius, r, is 10.

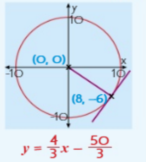



**EXAMPLE:** Find the equation of the tangent to $x^2 + y^2 = 100$ at the point (8, –6). (Grade 9)

1) Find the gradient of the line from the origin to (8, –6). This is a radius of the circle.
Gradient $= \frac{\text{Change in } y}{\text{Change in } x} = \frac{-6-0}{8-0} = \frac{-3}{4}$

2) A tangent meets a radius at 90°, (see p.76) so they are perpendicular — so the gradient of the tangent is $-\frac{1}{m}$.
Gradient of tangent $= -\frac{1}{m} = -\frac{1}{-\frac{3}{4}} = \frac{4}{3}$

3) Find the equation of the tangent by substituting (8, –6) into $y = mx + c$.
$y = mx + c \Rightarrow (-6) = \frac{4}{3}(8) + c$
$-6 = \frac{32}{3} + c$
$c = -\frac{50}{3}$




$y = \frac{4}{3}x - \frac{50}{3}$

## Harder Graphs

Before you leave this page, you should be able to close your eyes and picture these three graphs in your head, properly labelled and everything. If you can't, you need to learn them more. I'm not kidding.

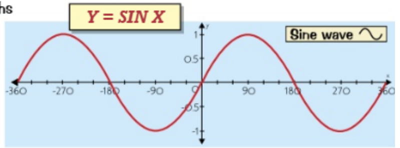
### Sine 'Waves' and Cos 'Buckets' (Grade 8)

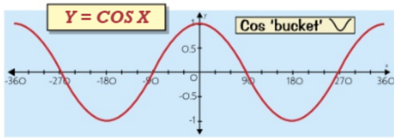
1) The underlying shape of the sin and cos graphs is identical — they both bounce between y-limits of exactly +1 and –1.

2) The only difference is that the sin graph is shifted right by 90° compared to the cos graph.

3) For 0° – 360°, the shapes you get are a Sine 'Wave' (one peak, one trough) and a Cos 'Bucket' (starts at the top, dips, and finishes at the top).

4) Sin and cos repeat every 360°. The key to drawing the extended graphs is to first draw the 0° – 360° cycle of either the Sine 'WAVE' or the Cos 'BUCKET' and then you can repeat it forever in both directions as shown above.

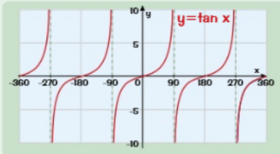



### Tan x can be Any Value at all (Grade 8)

tan x is different from sin x or cos x — it goes between $-\infty$ and $+\infty$.

**Tan x repeats every 180°**

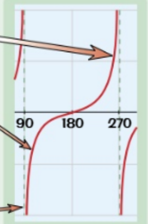



tan x is undefined at ±90°, ±270°,...

As you approach one of these undefined points from the left, tan x just shoots up to infinity.

As you approach from the right, it drops to minus infinity.

The graph never ever touches these lines. But it does get infinitely close, if you see what I mean...



tan x goes from $-\infty$ to $+\infty$ every 180°.

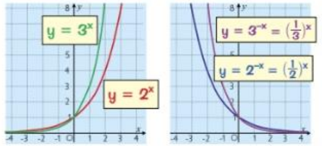
So it repeats every 180° and takes every possible value in each 180° interval.

The easiest way to sketch any of these graphs is to plot the important points which happen every 90° (e.g. –180°, –90°, 0°, 90°, 180°, 270°, 360°, 450°, 540°...) and then just join the dots up.

## Harder Graphs

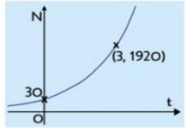
Here are two more graph types you need to be able to plot or sketch. Knowing what you're aiming for really helps.

### $k^x$ Graphs: $y = k^x$ or $y = k^{-x}$ (k is some positive number) (Grade 7)




1) These 'exponential' graphs are always above the x-axis, and always go through the point (0, 1).
2) If $k > 1$ and the power is +ve, the graph curves upwards.
3) If k is between 0 and 1 OR the power is negative, then the graph is flipped horizontally.

**EXAMPLE:** This graph shows how the number of victims of an alien virus (N) increases in a science fiction film. The equation of the graph is $N = fg^t$, where t is the number of days into the film. f and g are positive constants. Find the values of f and g.

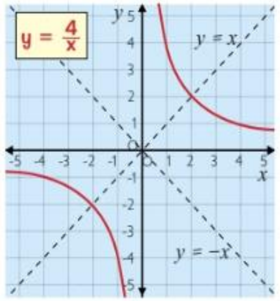



$g^0 = 1$, so you can find f.

When $t = 0$, $N = 30$ so substitute these values into the equation: $30 = fg^0 \Rightarrow 30 = f \times 1 \Rightarrow f = 30$

Substitute in $t = 3$, $N = 1920$: $N = 30g^t \Rightarrow 1920 = 30g^3$
$g = \sqrt[3]{64} \Rightarrow g = 4$

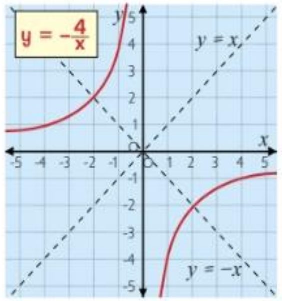
### 1/x (Reciprocal) Graphs: $y = A/x$ or $xy = A$ (Grade 6)




These are all the same basic shape, except the negative ones are in opposite quadrants to the positive ones (as shown). The two halves of the graph don't touch. The graphs don't exist for $x = 0$.

They're all symmetrical about the lines $y = x$ and $y = -x$.

(You get this type of graph with inverse proportion — see p.63)

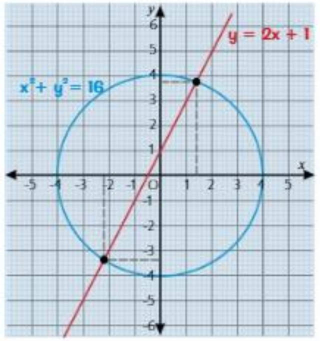



## Plot Both Graphs and See Where They Cross

**EXAMPLE:** By plotting the graphs, solve the simultaneous equations $x^2 + y^2 = 16$ and $y = 2x + 1$. (Grade 7)

1) DRAW BOTH GRAPHS.
$x^2 + y^2 = 16$ is the equation of a circle with centre (0, 0) and radius 4 (see p.49). Use a pair of compasses to draw it accurately.

2) LOOK FOR WHERE THE GRAPHS CROSS.
The straight line crosses the circle at two points. Reading the $x$ and $y$ values of these points gives the solutions $x = 1.4$, $y = 3.8$ and $x = -2.2$, $y = -3.4$ (all to 1 decimal place).



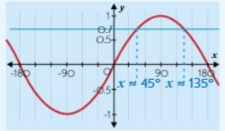

## Using Graphs to Solve Harder Equations

**EXAMPLES:**

1. The graph of $y = \sin x$ is shown to the right. Use the graph to estimate the solutions to $\sin x = 0.7$ between –180° and 180° (Grade 7)

Draw the line $y = 0.7$ on the graph, then read off where it crosses $\sin x$.

The solutions are $x \approx 45°$ and $x \approx 135°$.




2. The graph of $y = 2x^2 - 3x$ is shown on the right. (Grade 8)

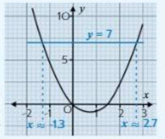
a) Use the graph to estimate both solutions to $2x^2 - 3x = 7$.

$2x^2 - 3x = 7$ is what you get when you put $y = 7$ into the equation:

1) Draw a line at $y = 7$.
2) Read the x-values where the curve crosses this line.

The solutions are around $x = -1.3$ and $x = 2.7$.

Quadratic equations usually have 2 solutions.




b) Find the equation of the line you would need to draw on the graph to solve $2x^2 - 5x + 1 = 0$

This is a bit nasty — the trick is to rearrange the given equation $2x^2 - 5x + 1 = 0$ so that you have $2x^2 - 3x$ (the graph) on one side.

$2x^2 - 5x + 1 = 0$
Adding $2x - 1$ to both sides: $2x^2 - 3x = 2x - 1$
So the line needed is $y = 2x - 1$.

The sides of this equation represent the two graphs $y = 2x^2 - 3x$ and $y = 2x - 1$. Finding the points where these graphs cross will give the solutions to $2x^2 - 5x + 1 = 0$