

Big idea: Ratio proportion and Rates of Change

Key skills:

- Multiplicative Reasoning
- Geometric Reasoning
- Algebraic Reasoning

Key Vocabulary

Proportion, Direct, Inverse, Varies, Best Buy, Scaling UP/Down,

Learn the Golden Rule for Proportion Questions

GRADE 3

There are lots of exam questions which at first sight seem completely different but in fact they can all be done using the **GOLDEN RULE**...

DIVIDE FOR ONE, THEN TIMES FOR ALL

EXAMPLE: 5 pints of milk cost £1.30. How much will 3 pints cost?

The **GOLDEN RULE** tells you to:

Divide the price by 5 to find how much **FOR ONE PINT**, then multiply by 3 to find how much **FOR 3 PINTS**.

$$1 \text{ pint: } £1.30 \div 5 = 0.26 = 26p$$

$$3 \text{ pints: } 26p \times 3 = 78p$$



EXAMPLE: Emma is handing out some leaflets. She gets paid per leaflet she hands out. If she hands out 300 leaflets she gets £2.40. How many leaflets will she have to hand out to earn £8.50?

Divide by £2.40 to find how many leaflets she has to hand out to earn £1.

$$\text{To earn } £1: 300 \div £2.40 = 125 \text{ leaflets}$$

Multiply by £8.50 to find how many leaflets she has to hand out to earn £8.50.

$$\text{To earn } £8.50: 125 \times £8.50 = 1062.5$$

So she'll need to hand out **1063** leaflets.

You need to round your answer up because 1062 wouldn't be enough.

Scaling Recipes Up or Down

GRADE 3

EXAMPLE: Judy is making orange and pineapple punch using the recipe shown on the right. She wants to make enough to serve 20 people. How much of each ingredient will Judy need?

Fruit Punch (serves 8)
800 ml orange juice
140 g fresh pineapple

The **GOLDEN RULE** tells you to divide each amount by 8 to find how much **FOR ONE PERSON**, then multiply by 20 to find how much **FOR 20 PEOPLE**.

So for 1 person you need:

And for 20 people you need:

$$800 \text{ ml} \div 8 = 100 \text{ ml orange juice} \Rightarrow 20 \times 100 \text{ ml} = 2000 \text{ ml orange juice}$$

$$140 \text{ g} \div 8 = 17.5 \text{ g pineapple} \Rightarrow 20 \times 17.5 \text{ g} = 350 \text{ g pineapple}$$

Best Buy Questions

GRADE 3

A slightly different type of direct proportion question is comparing the 'value for money' of 2 or 3 similar items. For these, follow the second **GOLDEN RULE**...

Divide by the **PRICE in pence** (to get the amount per penny)

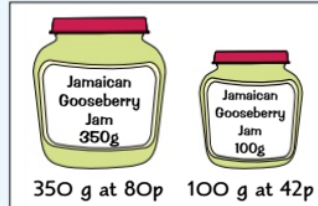
EXAMPLE: The local 'Supplies 'n' Vittals' stocks two sizes of Jamaican Gooseberry Jam, as shown on the right. Which of these represents better value for money?

Follow the **GOLDEN RULE** — divide by the price in pence to get the **amount per penny**.

$$\text{In the 350 g jar you get } 350 \text{ g} \div 80p = 4.38 \text{ g per penny}$$

$$\text{In the 100 g jar you get } 100 \text{ g} \div 42p = 2.38 \text{ g per penny}$$

The 350 g jar is better value for money, because you get more jam per penny.



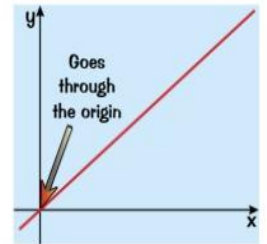
In some cases it might be easier to divide by the weight to get the **cost per gram**. If you're feeling confident then you can do it this way — if not, the golden rule **always works**.

Graphing Direct Proportion

GRADE 4

Two things are in direct proportion if, when you plot them on a graph, you get a straight line through the origin.

Remember, the **general equation** for a straight line through the origin is $y = Ax$ (see p.43) where A is a number. All direct proportions can be written as an equation in this form.



EXAMPLE: The amount of petrol, p litres, a car uses is directly proportional to the distance, d km, that the car travels. The car used 12 litres of petrol on a 160 km journey.

a) Write an equation in the form $p = Ad$ to represent this direct proportion.

$$1) \text{ Put the values of } p = 12 \text{ and } d = 160 \text{ into the equation to find the value of } A.$$

$$12 = A \times 160$$

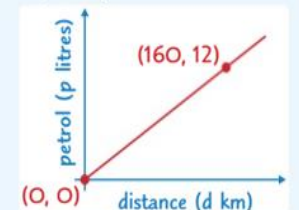
$$A = \frac{12}{160}$$

$$A = 0.075$$

2) Put the value of A **back into** the equation.

$$p = 0.075d$$

b) Sketch the graph of this direct proportion, marking two points on the line.



Knowledge Organiser: Mathematics

Year 11 Foundation Spring Term 1

Suggested websites: Maths Genie, Save My Exam and Corbett Maths



There's more about formula triangles on p69 if you need to jog your memory.

Big idea: Ratio proportion and Rates of Change

Key Vocabulary

Mass, Volume, Density, Sequence, Term, Sine, Cosine, Tangent, Pythagoras Theorem

Key skills:

- Multiplicative Reasoning
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Solving Inverse Proportion Questions

On page 58 you saw the 'divide and times' method for direct proportions. Well, inverse proportions are the opposite so you have to:

TIMES for ONE, then DIVIDE for ALL

EXAMPLES:

1. It takes 3 farmers 10 hours to plough a field. How long would it take 6 farmers?

Multiply by 3 to find how long it would take 1 farmer. $10 \times 3 = 30$ hours for 1 farmer

Divide by 6 to find how long it would take 6 farmers. $30 \div 6 = 5$ hours for 6 farmers

Note: Another way of looking at this question is that there are twice as many farmers, so it will take half as long ($10 \div 2 = 5$ hours).

2. 4 bakers can decorate 100 cakes in 5 hours.

If 5 bakers work at the same rate, how much quicker would they decorate 100 cakes?

Multiply by 4 to find how long it would take 1 baker. $5 \text{ hours} \times 4 = 20$ hours for 1 baker

Divide by 5 to find how long it would take 5 bakers. $20 \div 5 = 4$ hours for 5 bakers

So 5 bakers are $5 - 4 = 1$ hour quicker than 4.

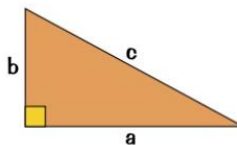
Pythagoras' Theorem is Used on Right-Angled Triangles

Pythagoras' theorem only works for RIGHT-ANGLED TRIANGLES.

It uses two sides to find the third side.

$$a^2 + b^2 = c^2$$

short sides long side



The trouble is, the formula can be quite difficult to use. Instead, it's a lot better to just remember these three simple steps, which work every time:

1) SQUARE THEM

SQUARE THE TWO NUMBERS that you are given, (use the \times button if you've got your calculator).

2) ADD or SUBTRACT

To find the **longest side**, **ADD** the two squared numbers. $a^2 + b^2 = c^2$
To find a **shorter side**, **SUBTRACT** the smaller from the larger. $c^2 - b^2 = a^2$

3) SQUARE ROOT

Once you've got your answer, take the **SQUARE ROOT** (use the $\sqrt{\quad}$ button on your calculator).

$$c = \sqrt{a^2 + b^2}$$

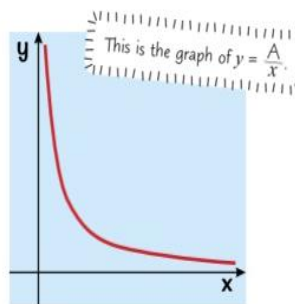
$$a = \sqrt{c^2 - b^2}$$

Graphing Inverse Proportion

When two things are in inverse proportion, one increases as the other decreases.

On the graph you can see that as the value of **x increases**, the value of **y decreases**. E.g. if x is **doubled**, y is **halved**, or if x is **multiplied by 5**, y is **divided by 5**.

The general equation for inverse proportion is $y = \frac{A}{x}$.



EXAMPLE:

Circle each of the equations below that show that s is inversely proportional to t.

$s = \frac{3}{t}$ $9s = t$ $t = \frac{1}{s}$ $s = \frac{3}{t^2}$ $s = \frac{3}{t} + 7$ $\frac{s}{5} = \frac{1}{t}$
 $s = \frac{3}{t}$ $s = \frac{1}{t}$ $s = \frac{5}{t}$

Check which equations can be written in the form $s = \frac{A}{t}$.

The 3 Trigonometry Formulas

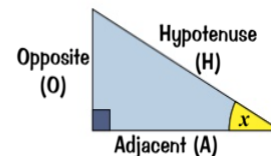
There are three basic trig formulas — each one links two sides and an angle of a right-angled triangle.

$$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan x = \frac{\text{Opposite}}{\text{Adjacent}}$$

- The **Hypotenuse** is the **LONGEST SIDE**.
- The **Opposite** is the side **OPPOSITE** the angle **being used** (x).
- The **Adjacent** is the (other) side **NEXT TO** the angle **being used**.



Deciding if a Term is in a Sequence

You might be given the nth term and asked if a **certain value** is in the sequence. The trick here is to **set the expression equal to that value** and solve to find n. If n is a **whole number**, the value is **in** the sequence.

EXAMPLE:

A sequence is given by the rule $6n - 2$.

a) Find the 6th term in the sequence.

Just put $n = 6$ into the expression:
 $(6 \times 6) - 2 = 36 - 2$
 $= 34$

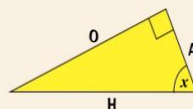
b) Is 45 a term in this sequence?

Set it equal to 45... $6n - 2 = 45$
 $6n = 47$...and solve for n.
 $n = 47 \div 6 = 7.8333...$
 n is not a whole number, so 45 is **not** in the sequence $6n - 2$.

Formula Triangles Make Things Easier

A great way to tackle trig questions is to convert the formulas into **formula triangles**. Then you can use the **same method every time**, no matter which side or angle is being asked for.

- 1) Label the three sides **O, A and H** (Opposite, Adjacent and Hypotenuse).
- 2) Write down '**SOH CAH TOA**'.
- 3) Decide which **two sides are involved**: O, H, A, H or O, A and choose **SOH, CAH or TOA** accordingly.
- 4) Turn the one you choose into a **FORMULA TRIANGLE**:



In the formula triangles, S represents sin x, C is cos x, and T is tan x.

5) **Cover up** the thing you want to find with your finger, and write down whatever is left showing.

6) **Stick in the numbers** and work it out using the **sin, cos and tan** buttons on your **calculator**.

If you're finding an **angle**, you'll need to add an extra step to find the **inverse**. See next page.

Finding the nth Term of a Sequence

This method works for sequences with a **common difference** — where you **add or subtract the same number** each time.

EXAMPLE:

Find an expression for the nth term of the sequence that starts 5, 8, 11, 14, ...

n:	1	2	3	4
term:	5	8	11	14
		+3	+3	+3
3n:	3	6	9	12
	+2	+2	+2	+2
term:	5	8	11	14

1 Find the **common difference**. It's 3, so this tells you '**3n**' is in the formula.

2 List the values of **3n**.

3 Work out what you have to **add or subtract** to get from 3n to the term. So it's **+2**.

4 Put '**3n**' and '**+2**' together.

So the expression for the nth term is $3n + 2$.

Check your formula by putting the first few values of n back in:

$n = 1$ gives $3n + 2 = 3 + 2 = 5$ ✓
 $n = 2$ gives $3n + 2 = 6 + 2 = 8$ ✓