Knowledge Organiser: Mathematics
Year 11 Foundation Autumn Term 2

Suggested websites: Maths Genie, Save My Exam and Corbett Maths

Smarks

Big idea: Algebra

Key skills:
- Expanding and Factorising
- Changing the Subject
- Functions

Key Vocabulary

Expand, Simplify, Coefficient, Factorise, Terms, Brackets, Like Terms, Quadratic

Multiplying Brackets (GRADE 3)

The key thing to remember about multiplying brackets is that the thing outside the brackets multiplies each separate term inside the brackets.

EXAMPLE: Expand the following:

a) $3(2x + 5)$
$= (3 \times 2x) + (3 \times 5)$
$= 6x + 15$

b) $-4(3y - 2)$
$= (-4 \times 3y) + (-4 \times -2)$
$= -12y + 8$

c) $2e(e - 4)$
$= (2e \times e) + (2e \times -4)$
$= 2e^2 - 8e$

EXAMPLE: Expand $x(2x + 1) + y(y - 4) + 3x(y + 2)$

1) Expand each bracket separately. $x(2x + 1) + y(y - 4) + 3x(y + 2)$
$= 2x^2 + x + y^2 - 4y + 3xy + 6x$

2) Group together like terms. $= 2x^2 + x + 6x + 3xy + y^2 - 4y$

3) Simplify the expression. $= 2x^2 + 7x + 3xy + y^2 - 4y$

D.O.T.S. — The Difference Of Two Squares (GRADE 5)

The 'difference of two squares' (D.O.T.S. for short) is where you have 'one thing squared' take away 'another thing squared'. There's a quick and easy way to factorise it — just use the rule below:

$$a^2 - b^2 = (a + b)(a - b)$$

EXAMPLE: Factorise:

a) $t^2 - 1$ Answer: $t^2 - 1 = (t + 1)(t - 1)$
Don't forget that 1 is a square number.

b) $s^2 - 64$ Answer: $s^2 - 64 = (s + 8)(s - 8)$
$64 = 8^2$, so in the formula above, $b = 8$.

c) $25m^2 - 1$ Answer: $25m^2 - 1 = (5m + 1)(5m - 1)$
Here you have to remember that it's 5m, not just m.

d) $9p^2 - 16q^2$ Answer: $9p^2 - 16q^2 = (3p + 4q)(3p - 4q)$
This time you had to spot that 9 and 16 are square numbers.

Use the FOIL Method to Multiply Out Double Brackets (GRADE 4)

There's a handy way to multiply double brackets — it's called the FOIL method and works like this:

First — multiply the first term in each bracket together
Outside — multiply the outside terms (i.e. the first term in the first bracket by the second term in the second bracket)
Inside — multiply the inside terms (i.e. the second term in the first bracket by the first term in the second bracket)
Last — multiply the second term in each bracket together

When multiplying double brackets, you get 4 terms — and 2 of them usually combine to leave 3 terms.

EXAMPLES:

1. Expand and simplify $(x + 3)(x + 8)$

$(x + 3)(x + 8) = (x \times x) + (x \times 8) + (3 \times x) + (3 \times 8)$ (F O I L)
$= x^2 + 8x + 3x + 24$
$= x^2 + 11x + 24$

The two x-terms combine together.

Factorising — Putting Brackets In (GRADE 4)

This is the exact reverse of multiplying out brackets. You have to look for common factors — numbers or letters that go into every term. Here's the method to follow:

1) Take out the biggest number that goes into all the terms.
2) For each letter in turn, take out the highest power (e.g. x, x^2, etc.) that will go into EVERY term.
3) Open the bracket and fill in all the bits needed to reproduce each term.
4) Check your answer by multiplying out the bracket and making sure it matches the original expression.

EXAMPLES:

1. Factorise $8y + 4$
Biggest number that goes into 8 and 4
Decide what you need to multiply 4 by to get 8y and 4.
$4(2y + 1)$
Check: $4(2y + 1) = 8y + 4$ ✓

2. Factorise $3x^2 + 6x$
Biggest number that goes into 3 and 6
Highest power of x that will go into both terms
$3x(x + 2)$
Check: $3x(x + 2) = 3x^2 + 6x$ ✓

REMEMBER: The bits taken out and put at the front are the common factors. The bits inside the bracket are what's needed to get back to the original terms if you multiply the bracket out again.

Write Out Squared Brackets as Double Brackets (GRADE 4)

Always write out squared brackets as two brackets (to avoid mistakes) — then multiply them out using the FOIL method above.

EXAMPLE: Expand and simplify $(3x + 2)^2$

$(3x + 2)^2 = (3x + 2)(3x + 2)$ ← Write out the expression as two brackets, then use the FOIL method.
$= 9x^2 + 6x + 6x + 4$
$= 9x^2 + 12x + 4$

DON'T make the mistake of thinking that $(3x + 2)^2 = 9x^2 + 4$ (this is wrong wrong wrong).

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Key Vocabulary

Input, Output, Expression, Coefficient, Evaluate, Functions, Inverse Substitute, Rearrange, Solving,

Functions Produce Outputs from Inputs

GRADE 2

1) A function takes an input, processes it (e.g. multiplies it by 5 and adds 2) and outputs a value.
2) If you have to use a function machine, just put in the number, follow the steps and see what comes ou
3) If you're given the output and have to find the input, use the function machine in reverse.

EXAMPLE: The function machine below represents the function 'multiply by 5 and add 2'.

If this was an equation, it'd be written as $y = 5x + 2$.

a) Find the value of y when $x = 5$.
Just put 5 into the machine: $5 \xrightarrow{\times 5} 25 \xrightarrow{+2} 27$. So $y = 27$.

b) Find the value of x when $y = 42$.
This time, put $y = 42$ into the machine and work backwards:
$42 \xrightarrow{-2} 40 \xrightarrow{\div 5} 8$. So $x = 8$.

Don't forget to reverse each step as well — so +2 becomes −2 and ×5 becomes ÷5.

EXAMPLE: Rearrange $m = \frac{n}{4} - 7$ to make n the subject of the formula.

$m = \frac{n}{4} - 7$ — The opposite of −7 is +7, so add 7 to both sides.

$(+7)$ $m + 7 = \frac{n}{4} - 7 + 7$

$m + 7 = \frac{n}{4}$ — The opposite of ÷4 is ×4, so multiply both sides by 4.

$(\times 4)$ $(m + 7) \times 4 = \frac{n}{4} \times 4$

$4(m + 7) = n$ OR $n = 4m + 28$

Changing the Subject of a Formula

GRADE 5

Rearranging formulas means making a different letter the subject, e.g. getting 'y = ' from 'x = 3y + 2'. Fortunately, you can use the same methods that you used for solving equations (see p29-30) — here's a quick reminder:

Golden Rules
1) Always do the SAME thing to both sides of the formula.
2) To get rid of something, do the opposite.
The opposite of + is − and the opposite of − is +.
The opposite of × is ÷ and the opposite of ÷ is ×.
3) Keep going until you have the letter you want on its own.

EXAMPLE: Rearrange $a = 3b + 4$ to make b the subject of the formula.

$a = 3b + 4$ — The opposite of +4 is −4, so take away 4 from both sides.

(-4) $a - 4 = 3b + 4 - 4$

$a - 4 = 3b$ — The opposite of ×3 is ÷3, so divide both sides by 3.

$(\div 3)$ $(a - 4) \div 3 = 3b \div 3$

Careful here — you divide the whole side by 3, not just one term.

$\frac{a-4}{3} = b$ OR $b = \frac{a-4}{3}$

EXAMPLE: Rearrange $p = 5(q + 2)$ to make q the subject of the formula.

$p = 5q + 10$ — Multiply out the brackets.

(-10) $p - 10 = 5q + 10 - 10$ — The opposite of +10 is −10, so take away 10 from both sides.

$p - 10 = 5q$

$(\div 5)$ $(p - 10) \div 5 = 5q \div 5$ — The opposite of ×5 is ÷5, so divide both sides by 5.

$\frac{p-10}{5} = q$ OR $q = \frac{p-10}{5}$