Knowledge Organiser: Mathematics
Year 11 Foundation Summer Term 1

Suggested websites: Maths Genie, Save My Exam and Corbett Maths

Big idea: Number, Algebra, Geometry and Measures, and Ratio, Proportion and Rates of change

Revision Lessons

Powers

Four Easy Rules:

1) When MULTIPLYING, you ADD THE POWERS. e.g. $3^4 \times 3^6 = 3^{4+6} = 3^{10}$
2) When DIVIDING, you SUBTRACT THE POWERS. e.g. $c^4 \div c^2 = c^{4-2} = c^2$
3) When RAISING one power to another, you MULTIPLY THE POWERS. e.g. $(3^2)^4 = 3^{2\times4} = 3^8$
4) FRACTIONS — Apply the power to both TOP and BOTTOM. e.g. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

EXAMPLE: $a = 5^9$ and $b = 5^4 \times 5^2$. What is the value of $\frac{a}{b}$?

1) Work out b — add the powers: $b = 5^4 \times 5^2 = 5^{4+2} = 5^6$
2) Divide a by b — subtract the powers: $\frac{a}{b} = 5^9 \div 5^6 = 5^{9-6} = 5^3 = 125$

One Trickier Rule

To find a negative power — turn it upside-down.

People have real difficulty remembering this — whenever you see a negative power you need to immediately think: "Aha, that means turn it the other way up and make the power positive".

E.g. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$

Speed = Distance ÷ Time

Speed is the distance travelled per unit time — the number of km per hour or metres per second.

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} \qquad \text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}} \qquad \text{DISTANCE} = \text{SPEED} \times \text{TIME}$$

A formula triangle is a mighty handy tool for remembering formulas. Here's the one for speed.
To remember the order of the letters (S^DT) we have the words SaD Times.
So if it's a question on speed, distance and time, just say SAD TIMES.

HOW DO YOU USE FORMULA TRIANGLES?
1) COVER UP the thing you want to find and WRITE DOWN what's left.
2) Now PUT IN THE VALUES for the other two things and WORK IT OUT.

E.g. to get the formula for speed from the triangle, cover up S and you're left with $\frac{D}{T}$.

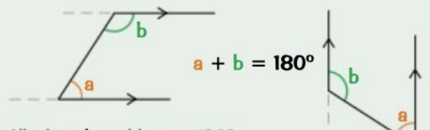
*Alternate, Allied and Correspondi*ng *Angles*

Watch out for these 'Z', 'C', 'U' and 'F' shapes popping up.
They're a dead giveaway that you've got a pair of parallel lines.

Dor

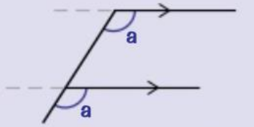
need to use their proper names

ALLIED ANGLES



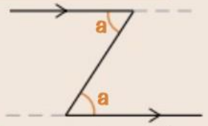
Allied angles add up to 180°.
They are found in a C- or U-shape.

CORRESPONDING ANGLES



Corresponding angles are the same.
They are found in an F-shape.

ALTERNATE ANGLES

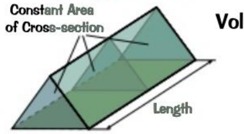


Alternate angles are the same.
They are found in a Z-shape.

Volumes of Prisms

A PRISM is a solid (3D) object which is the same shape all the way through — i.e. it has a CONSTANT AREA OF CROSS-SECTION.

Triangular Prism

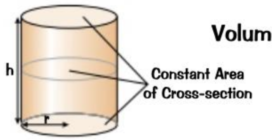


Volume of Prism = cross-sectional area × length

$V = A \times L$

This formula works for any prism.

Cylinder
(circular prism)



Volume of Cylinder = area of circle × height

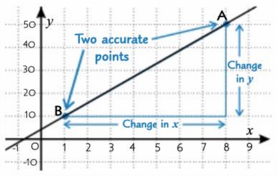
$V = \pi r^2 h$

Finding the Gradient

Straight-Line Graphs — Gradients

The gradient of a line is a measure of its slope. The bigger the number, the steeper the line.

EXAMPLE: Find the gradient of the straight line shown.



1. Find two accurate points and complete the triangle.

Choose easy points with positive coordinates.

Two points that can be read accurately are:
Point A: (8, 50) Point B: (1, 10)

2. Find the change in y and the change in x.

Change in $y = 50 - 10 = 40$
Change in $x = 8 - 1 = 7$

Make sure you subtract the x-coordinates the SAME WAY ROUND as you do the y-coordinates.
E.g. y-coord. of pt A – y-coord. of pt B and x-coord. of pt A – x-coord. of pt B

3. LEARN this formula, and use it:

$$\text{GRADIENT} = \frac{\text{CHANGE IN Y}}{\text{CHANGE IN X}}$$

Gradient = $\frac{40}{7}$ = 5.71 (to 2 d.p.)

Make sure you get the formula the right way up.
Remember it's VERy HOt — VERtical over HOrizontal.

4. Check the sign's right.

If it slopes uphill left → right (↗) then it's positive.
If it slopes downhill left → right (↘) then it's negative.

As the graph goes uphill, the gradient is positive. So the gradient is 5.71 (not -5.71).

Revision Topics And key skils

Review indices and index law

Review angles in parallel lines.

Review speed, distance and time.

Review Trig ratios

Review Pythagoras' theorem.

Review transformation.

Review volume of prisms

Review gradient and rate of change

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Revision Lessons

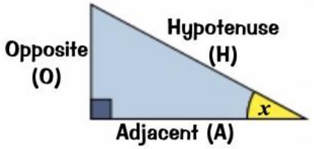
Trigonometry — Sin, Cos, Tan

The 3 Trigonometry Formulas

There are three basic trig formulas — each one links two sides and an angle of a right-angled triangl

$$\text{Sin } x = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cos } x = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{Tan } x = \frac{\text{Opposite}}{\text{Adjacent}}$$

- The Hypotenuse is the LONGEST SIDE.
- The Opposite is the side OPPOSITE the angle being used (x).
- The Adjacent is the (other) side NEXT TO the angle being used.

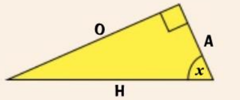


Formula Triangles Make Things Easier

There's more about formula triangles on p.69 if you need to jog your memory.

A great way to tackle trig questions is to convert the formulas into formula triangles. Then you can use the same method every time, no matter which side or angle is being asked for.

1) Label the three sides O, A and H (Opposite, Adjacent and Hypotenuse).
2) Write down 'SOH CAH TOA'.
3) Decide which two sides are involved: O,H A,H or O,A and choose SOH, CAH or TOA accordingly.



4) Turn the one you choose into a FORMULA TRIANGLE:

In the formula triangles, S represents sin x, C is cos x, and T is tan x.

5) Cover up the thing you want to find with your finger, and write down whatever is left showing.
6) Stick in the numbers and work it out using the sin, cos and tan buttons on your calculator.

If you're finding an angle, you'll need to add an extra step to find the inverse. See next page.

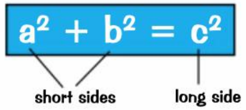
Pythagoras' Theorem

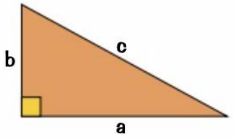
Pythagoras' Theorem is Used on Right-Angled Triangles

Pythagoras' theorem only works for RIGHT-ANGLED TRIANGLES.
It uses two sides to find the third side.

The formula for Pythagoras' theorem is:

$$a^2 + b^2 = c^2$$





The trouble is, the formula can be quite difficult to use. Instead, it's a lot better to just remember these three simple steps, which work every time:

1) **SQUARE THEM** — SQUARE THE TWO NUMBERS that you are given, (use the x^2 button if you've got your calculator.)

2) **ADD or SUBTRACT** — To find the longest side, ADD the two squared numbers. $a^2 + b^2 = c^2$
To find a shorter side, SUBTRACT the smaller from the larger. $c^2 - b^2 = a^2$

3) **SQUARE ROOT** — Once you've got your answer, take the SQUARE ROOT (use the √ button on your calculator).
$c = \sqrt{a^2 + b^2}$
$a = \sqrt{c^2 - b^2}$

The Four Transformations

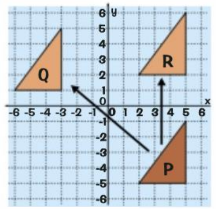
1) Translations

In a translation, the amount the shape moves by is given as a vector (see p.103-104) written $\begin{pmatrix} x \\ y \end{pmatrix}$ — where x is the horizontal movement (i.e. to the right) and y is the vertical movement (i.e. up). If the shape moves left and down, x and y will be negative.

EXAMPLE:
a) Describe the transformation that maps triangle P onto Q.
b) Describe the transformation that maps triangle P onto R.

a) To get from P to Q, you need to move 8 units left and 6 units up, so...
The transformation from P to Q is a translation by the vector $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$.

b) The transformation from P to R is a translation by the vector $\begin{pmatrix} 0 \\ 7 \end{pmatrix}$.



2) Rotations

To describe a rotation, you must give 3 details:
1) The angle of rotation (usually 90° or 180°).
2) The direction of rotation (clockwise or anticlockwise).
3) The centre of rotation (often, but not always, the origin).

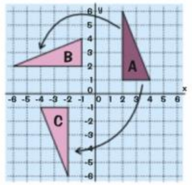
For a rotation of 180°, it doesn't matter whether you go clockwise or anticlockwise.

EXAMPLE:
a) Describe the transformation that maps triangle A onto B.
b) Describe the transformation that maps triangle A onto C.

a) The transformation from A to B is a rotation of 90° anticlockwise about the origin.

b) The transformation from A to C is a rotation of 180° clockwise (or anticlockwise) about the origin.

If it helps, you can use tracing paper to help you find the centre of rotation.

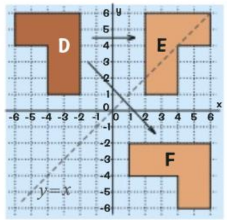


3) Reflections

For a reflection, you must give the equation of the mirror line.

EXAMPLE:
a) Describe the transformation that maps shape D onto shape E.
b) Describe the transformation that maps shape D onto shape F.

a) The transformation from D to E is a reflection in the y-axis.
b) The transformation from D to F is a reflection in the line $y = x$.



4) Enlargements

For an enlargement, you must specify:

1) The scale factor.
2) The centre of enlargement.

$$\text{scale factor} = \frac{\text{new length}}{\text{old length}}$$

1) The scale factor for an enlargement tells you how long the sides of the new shape are compared to the old shape. E.g. a scale factor of 3 means you multiply each side length by 3.
2) If you're given the centre of enlargement, then it's vitally important where your new shape is on the grid.

The scale factor tells you the RELATIVE DISTANCE of the old points and new points from the centre of enlargement.

So, a scale factor of 2 means the corners of the enlarged shape are twice as far from the centre of enlargement as the corners of the original shape.