Knowledge Organiser: Mathematics Year 11 Higher Autumn Term 2

Suggested websites: Maths Genie, Save My Exam and Corbett Maths

Big idea: Algebra

Key skills:
- Expanding and Factorising
- Changing the Subject
- Functions

Key Vocabulary

Expand, Simplify, Coefficient, Factorise, Terms, Brackets, Like Terms

Double Brackets (Grade 4)

have been reversed.

Double brackets are trickier than single brackets — this time, you have to multiply everything in the first bracket by everything in the second bracket. You'll get 4 terms, and usually 2 of them will combine to leave 3 terms. There's a handy way to multiply out double brackets — it's called the FOIL method:

First — multiply the first term in each bracket together
Outside — multiply the outside terms (i.e. the first term in the first bracket by the second term in the second bracket)
Inside — multiply the inside terms (i.e. the second term in the first bracket by the first term in the second bracket)
Last — multiply the second term in each bracket together

EXAMPLE: Expand and simplify $(2p - 4)(3p + 1)$

$(2p - 4)(3p + 1) = (2p \times 3p) + (2p \times 1) + (-4 \times 3p) + (-4 \times 1)$
$= 6p^2 + 2p - 12p - 4$
$= 6p^2 - 10p - 4$

The two p terms combine together.

Always write out SQUARED BRACKETS as TWO BRACKETS (to avoid mistakes), then multiply out as above.
So $(3x + 5)^2 = (3x + 5)(3x + 5) = 9x^2 + 15x + 15x + 25 = 9x^2 + 30x + 25$.
(DON'T make the mistake of thinking that $(3x + 5)^2 = 9x^2 + 25$ — this is wrong wrong wrong.)

D.O.T.S. — The Difference Of Two Squares (Grade 6)

The 'difference of two squares' (D.O.T.S. for short) is where you have 'one thing squared' take away 'another thing squared'. There's a quick and easy way to factorise it — just use the rule below:

$$a^2 - b^2 = (a + b)(a - b)$$

EXAMPLE: Factorise:

a) $9p^2 - 16q^2$ Answer: $9p^2 - 16q^2 = (3p + 4q)(3p - 4q)$
Here you had to spot that 9 and 16 are square numbers.

b) $3x^2 - 75y^2$ Answer: $3x^2 - 75y^2 = 3(x^2 - 25y^2) = 3(x + 5y)(x - 5y)$
This time, you had to take out a factor of 3 first.

c) $x^2 - 5$ Answer: $x^2 - 5 = (x + \sqrt{5})(x - \sqrt{5})$
Although 5 isn't a square number, you can write it as $(\sqrt{5})^2$.

Watch out — the difference of two squares can creep into other algebra questions. A popular exam question is to put a difference of two squares on the top or bottom of a fraction and ask you to simplify it. There's more on algebraic fractions on p.30.

EXAMPLE: Simplify $\frac{x^2 - 36}{5x + 30}$

The numerator is a difference of two squares.

$\frac{x^2 - 36}{5x + 30} = \frac{(x + 6)(x - 6)}{5(x + 6)} = \frac{x - 6}{5}$

Factorise the denominator.

Triple Brackets (Grade 7)

1) For three brackets, just multiply two together as above, then multiply the result by the remaining bracket.

It doesn't matter which pair of brackets you multiply together first.

2) If you end up with three terms in one bracket, you won't be able to use FOIL. Instead, you can reduce it to a series of single bracket multiplications — like in the example below.

EXAMPLE: Expand and simplify $(x + 2)(x + 3)(2x - 1)$

$(x + 2)(x + 3)(2x - 1) = (x + 2)(2x^2 + 5x - 3) = x(2x^2 + 5x - 3) + 2(2x^2 + 5x - 3)$
$= (2x^3 + 5x^2 - 3x) + (4x^2 + 10x - 6)$
$= 2x^3 + 9x^2 + 7x - 6$

Factorising — Putting Brackets In (Grade 4)

This is the exact reverse of multiplying out brackets. Here's the method to follow:

1) Take out the biggest number that goes into all the terms.
2) For each letter in turn, take out the highest power (e.g. x, x^2 etc.) that will go into EVERY term.
3) Open the bracket and fill in all the bits needed to reproduce each term.
4) Check your answer by multiplying out the bracket and making sure it matches the original expression.

EXAMPLES:

1. Factorise $3x^2 + 6x$
Biggest number that'll divide into 3 and 6. Highest power of x that will go into both terms.
$3x(x + 2)$
Check: $3x(x + 2) = 3x^2 + 6x$ ✓

2. Factorise $8x^2y + 2xy^2$
Biggest number that'll divide into 8 and 2. Highest powers of x and y that will go into both terms.
$2xy(4x + y)$
Check: $2xy(4x + y) = 8x^2y + 2xy^2$ ✓

REMEMBER: The bits taken out and put at the front are the common factors. The bits inside the bracket are what's needed to get back to the original terms if you multiply the bracket out again.

Rearrange Formulas with the Solving Equations Method (Grade 5)

Rearranging formulas is remarkably similar to solving equations. The method below is identical to the method for solving equations, except that I've added an extra step at the start.

1) Get rid of any square root signs by squaring both sides.
2) Get rid of any fractions.
3) Multiply out any brackets.
4) Collect all the subject terms on one side and all non-subject terms on the other.
5) Reduce it to the form '$Ax = B$' (by combining like terms). You might have to do some factorising here too.
6) Divide both sides by A to give '$x =$ '.
7) If you're left with '$x^2 =$ ', square root both sides to get '$x = \pm$ ' (don't forget the $\pm$).

x is the subject term here. A and B could be numbers or letters (or a mix of both).

What To Do If...

...the Subject Appears in a Fraction (Grade 5)

You won't always need to use all 7 steps in the method above — just ignore the ones that don't apply.

EXAMPLE: Make b the subject of the formula $a = \frac{5b + 3}{4}$.

There aren't any square roots, so ignore step 1.

2) Get rid of any fractions. (by multiplying every term by 4, the denominator) $(\times 4)$ $4a = \frac{4(5b + 3)}{4}$
$4a = 5b + 3$

There aren't any brackets so ignore step 3.

4) Collect all the subject terms on one side and all non-subject terms on the other. (remember that you're trying to make b the subject) (-3) $5b = 4a - 3$

5) It's now in the form $Ab = B$. (where A = 5 and B = 4a – 3)

6) Divide both sides by 5 to give 'b = '. $(\div 5)$ $b = \frac{4a - 3}{5}$

b isn't squared, so you don't need step 7.

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Key Vocabulary

Quadratic, Expression, Coefficient, Evaluate, Functions, Substitute,

Factorising a Quadratic (Grade 5)

1) 'Factorising a quadratic' means 'putting it into 2 brackets'.
2) The standard format for quadratic equations is: $ax^2 + bx + c = 0$.
3) If $a = 1$, the quadratic is much easier to deal with. E.g. $x^2 + 3x + 2 = 0$
4) As well as factorising a quadratic, you might be asked to solve the equation. This just means finding the values of x that make each bracket 0 (see example below).

See next page for when 'a' is not 1.

Factorising Method when a = 1 (Grade 5)

1) ALWAYS rearrange into the STANDARD FORMAT: $x^2 + bx + c = 0$.
2) Write down the TWO BRACKETS with the x's in: $(x \quad)(x \quad) = 0$.
3) Then find 2 numbers that MULTIPLY to give 'c' (the end number) but also ADD/SUBTRACT to give 'b' (the coefficient of x).
4) Fill in the +/– signs and make sure they work out properly.
5) As an ESSENTIAL CHECK, expand the brackets to make sure they give the original equation.
6) Finally, SOLVE THE EQUATION by setting each bracket equal to 0.

Ignore any minus signs at this stage.

You only need to do step 6) if the question asks you to solve the equation — if it just tells you to factorise, you can stop at step 5).

...there's a Square or Square Root Involved (Grade 5)

If the subject appears as a square or in a square root, you'll have to use steps 1 and 7 (not necessarily both).

EXAMPLE: Make u the subject of the formula $v^2 = u^2 + 2as$.

There aren't any square roots, fractions or brackets so ignore steps 1-3 (this is pretty easy so far).

4) Collect all the subject terms on one side and all non-subject terms on the other.
$(-2as)$ $u^2 = v^2 - 2as$

5) It's now in the form $Au^2 = B$ (where A = 1 and $B = v^2 - 2as$)

This is a real-life equation — v = final velocity, u = initial velocity, a = acceleration and s = displacement.

A = 1, which means it's already in the form '$u^2 =$ ', so ignore step 6.

7) Square root both sides to get '$u = \pm$ '. $(\sqrt{\ })$ $u = \pm\sqrt{v^2 - 2as}$

EXAMPLE: Make n the subject of the formula $2(m + 3) = \sqrt{n + 5}$.

1) Get rid of any square roots by squaring both sides.
$[2(m + 3)]^2 = (\sqrt{n + 5})^2$
$4(m^2 + 6m + 9) = n + 5$
$4m^2 + 24m + 36 = n + 5$

There aren't any fractions so ignore step 2.
The brackets were removed when squaring so ignore step 3.

4) Collect all the subject terms on one side and all non-subject terms on the other.
(-5) $n = 4m^2 + 24m + 31$ This is in the form 'n = ' so you don't need to do steps 5-7.

...the Subject Appears Twice (Grade 6)

Go home and cry. No, not really — you'll just have to do some factorising, usually in step 5.

EXAMPLE: Make p the subject of the formula $q = \frac{p+1}{p-1}$.

There aren't any square roots so ignore step 1.

2) Get rid of any fractions. $q(p - 1) = p + 1$
3) Multiply out any brackets. $pq - q = p + 1$
4) Collect all the subject terms on one side and all non-subject terms on the other.
$pq - p = q + 1$
5) Combine like terms on each side of the equation. $p(q - 1) = q + 1$

This is where you factorise — p was in both terms on the LHS so it comes out as a common factor.

6) Divide both sides by (q – 1) to give 'p = '. $p = \frac{q+1}{q-1}$ (p isn't squared, so you don't need step 7.)

When 'a' is Not 1 (Grade 7)

The basic method is still the same but it's a bit messier — the initial brackets are different as the first terms in each bracket have to multiply to give 'a'. This means finding the other numbers to go in the brackets is harder as there are more combinations to try. The best way to get to grips with it is to have a look at an example.

EXAMPLE: Solve $3x^2 + 7x - 6 = 0$.

1) $3x^2 + 7x - 6 = 0$
2) $(3x \quad)(x \quad) = 0$
3) Number pairs: 1 × 6 and 2 × 3

$(3x \quad 1)(x \quad 6)$ multiplies to give 18x and 1x which add/subtract to give 17x or 19x
$(3x \quad 6)(x \quad 1)$ multiplies to give 3x and 6x which add/subtract to give 9x or 3x
$(3x \quad 3)(x \quad 2)$ multiplies to give 6x and 3x which add/subtract to give 9x or 3x
$(3x \quad 2)(x \quad 3)$ multiplies to give 9x and 2x which add/subtract to give 11x or 7x ✓

$(3x \quad 2)(x \quad 3)$

4) $(3x - 2)(x + 3)$
5) $(3x - 2)(x + 3) = 3x^2 + 9x - 2x - 6$
$= 3x^2 + 7x - 6$ ✓
6) $(3x - 2) = 0 \Rightarrow x = \frac{2}{3}$
$(x + 3) = 0 \Rightarrow x = -3$

1) Rearrange into the standard format.
2) Write down the initial brackets — this time, one of the brackets will have a 3x in it.
3) The tricky part: first, find pairs of numbers that multiply to give c (= 6), ignoring the minus sign for now.
Then, try out the number pairs you just found in the brackets until you find one that gives 7x. But remember, each pair of numbers has to be tried in 2 positions (as the brackets are different — one has 3x in it).
4) Now fill in the +/– signs so that 9 and 2 add/subtract to give +7 (= b).
5) ESSENTIAL check — EXPAND the brackets.
6) SOLVE THE EQUATION by setting each bracket equal to 0 (if a isn't 1, one of your answers will be a fraction).

Functions

A function takes an input, processes it and outputs a value. There are two main ways of writing a function: $f(x) = 5x + 2$ or $f: x \to 5x + 2$. Both of these say 'the function f takes a value for x, multiplies it by 5 and adds 2'. Functions can look a bit scary-mathsy, but they're just like equations but with y replaced by f(x).

Evaluating Functions (Grade 6)

This is easy — just shove the numbers into the function and you're away.

EXAMPLE: $f(x) = x^2 - x + 7$. Find a) f(3) and b) f(–2)

a) $f(3) = (3)^2 - (3) + 7 = 9 - 3 + 7 = 13$
b) $f(-2) = (-2)^2 - (-2) + 7 = 4 + 2 + 7 = 13$

Combining Functions (Grade 8)

1) You might get a question with two functions, e.g. f(x) and g(x), combined into a single function (called a composite function).
2) Composite functions are written e.g. fg(x), which means 'do g first, then do f' — you always do the function closest to x first.
3) To find a composite function, rewrite fg(x) as f(g(x)), then replace g(x) with the expression it represents and then put this into f.

Watch out — usually fg(x) ≠ gf(x). Never assume that they're the same.

EXAMPLE: If $f(x) = 2x - 10$ and $g(x) = -\frac{x}{2}$, find: a) fg(x) and b) gf(x).

a) $fg(x) = f(g(x)) = f(-\frac{x}{2}) = 2(-\frac{x}{2}) - 10 = -x - 10$
b) $gf(x) = g(f(x)) = g(2x - 10) = -\left(\frac{2x - 10}{2}\right) = -(x - 5) = 5 - x$

Inverse Functions (Grade 8)

The inverse of a function f(x) is another function, $f^{-1}(x)$, which reverses f(x). Here's the method to find it:

1) Write out the equation $x = f(y)$
2) Rearrange the equation to make y the subject.
3) Finally, replace y with $f^{-1}(x)$.

f(y) is just the expression f(x), but with y's instead of x's

EXAMPLE: If $f(x) = \frac{12 + x}{3}$, find $f^{-1}(x)$.

1) Write out $x = f(y)$: $x = \frac{12 + y}{3}$
2) Rearrange to make y the subject: $3x = 12 + y$
$y = 3x - 12$
3) Replace y with $f^{-1}(x)$: $f^{-1}(x) = 3x - 12$

So here you just rewrite the function replacing f(x) with x and x with y.

You can check your answer by seeing if $f^{-1}(x)$ does reverse f(x): e.g. $f(9) = \frac{21}{3} = 7$, $f^{-1}(7) = 21 - 12 = 9$