Knowledge Organiser: Mathematics
Year 11 Higher Spring Term 1

Suggested websites: Maths Genie, Save My Exam and Corbett Maths

Big idea: Ratio proportion and Rates of Change

Key skills:
- Multiplicative Reasoning
- Geometric Reasoning
- Algebraic Reasoning

Key Vocabulary

Proportion, Direct, Inverse, Varies, Speed, Distance, Time,

Direct Proportion

1) Two quantities, A and B, are in direct proportion (or just in proportion) if increasing one increases the other one proportionally. So if quantity A is doubled (or trebled, halved, etc.), so is quantity B.
2) Remember this golden rule for direct proportion questions:

DIVIDE for ONE, then TIMES for ALL

EXAMPLE: Hannah pays £3.60 per 400 g of cheese. She uses 220 g of cheese to make 4 cheese pasties. How much would the cheese cost if she wanted to make 50 cheese pasties?

In 1 pasty there is:	$220 \text{ g} \div 4 = 55$ g of cheese
So in 50 pasties there is:	$55 \text{ g} \times 50 = 2750$ g of cheese
1 g of cheese would cost:	$£3.60 \div 400 = 0.9$p
So 2750 g of cheese would cost:	$0.9 \times 2750 = 2475\text{p} = £24.75$

There will often be lots of stages to direct proportion questions — keep track of what you've worked out at each stage.

Inverse Proportion

1) Two quantities, C and D, are in inverse proportion if increasing one quantity causes the other quantity to decrease proportionally. So if quantity C is doubled (or tripled, halved, etc.), quantity D is halved (or divided by 3, doubled etc.).
2) The rule for finding inverse proportions is:

TIMES for ONE, then DIVIDE for ALL

EXAMPLE: 4 bakers can decorate 100 cakes in 5 hours.

a) How long would it take 10 bakers to decorate the same number of cakes?

100 cakes will take 1 baker:	$5 \times 4 = 20$ hours
So 100 cakes will take 10 bakers:	$20 \div 10 = 2$ hours for 10 bakers

b) How long would it take 11 bakers to decorate 220 cakes?

100 cakes will take 1 baker:	20 hours
1 cake will take 1 baker:	$20 \div 100 = 0.2$ hours
220 cakes will take 1 baker:	$0.2 \times 220 = 44$ hours
220 cakes will take 11 bakers:	$44 \div 11 = 4$ hours

The number of bakers is inversely proportional to number of hours — but the number of cakes is directly proportional to the number of hours.

Types of Proportion

$\propto$ means 'is proportional to'.

1) The simple proportions are 'y is proportional to x' ($y \propto x$) and 'y is inversely proportional to x' ($y \propto \frac{1}{x}$).
2) You can always turn a proportion statement into an equation by replacing '$\propto$' with '$= k$' like this:

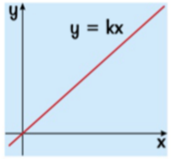
	Proportionality	Equation
'y is proportional to x'	$y \propto x$	$y = kx$
'y is inversely proportional to x'	$y \propto \frac{1}{x}$	$y = \frac{k}{x}$

k is just some constant (unknown number)

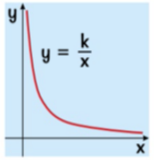
3) Trickier proportions involve y varying proportionally or inversely to some function of x, e.g. x^2, x^3, $\sqrt{x}$ etc

	Proportionality	Equation
'y is proportional to the square of x'	$y \propto x^2$	$y = kx^2$
't is proportional to the square root of h'	$t \propto \sqrt{h}$	$t = k\sqrt{h}$
'V is inversely proportional to r cubed'	$V \propto \frac{1}{r^3}$	$V = \frac{k}{r^3}$

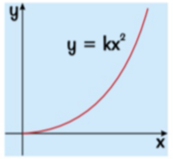
4) Once you've written the proportion statement as an equation you can easily graph it.



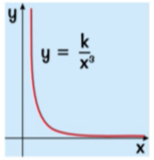
y is proportional to x



y is inversely proportional to x



y is proportional to x^2



y is inversely proportional to x^3

Handling Algebra Questions on Proportion

1) Write the sentence as a proportionality and replace '$\propto$' with '$= k$' to make an equation (as above).
2) Find a pair of values (x and y) somewhere in the question — substitute them into the equation to find k.
3) Put the value of k into the equation and it's now ready to use, e.g. $y = 3x^2$.
4) Inevitably, they'll ask you to find y, having given you a value for x (or vice versa).

EXAMPLE: G is inversely proportional to the square root of H. When G = 2, H = 16. Find an equation for G in terms of H, and use it to work out the value of G when H = 36.

1) Convert to a proportionality and replace $\propto$ with '$= k$' to form an equation. $G \propto \frac{1}{\sqrt{H}}$ $\quad G = \frac{k}{\sqrt{H}}$
2) Use the values of G and H (2 and 16) to find k. $2 = \frac{k}{\sqrt{16}} = \frac{k}{4} \Rightarrow k = 8$
3) Put the value of k back into the equation. $G = \frac{8}{\sqrt{H}}$ — This is the equation for G in terms of H.
4) Use your equation to find the value of G. When H = 36, $G = \frac{8}{\sqrt{36}} = \frac{8}{6} = \frac{4}{3}$

Speed = Distance ÷ Time

Speed is the distance travelled per unit time, e.g. the number of km per hour or metres per second.

$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}} \qquad \text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}} \qquad \text{DISTANCE} = \text{SPEED} \times \text{TIME}$$

Formula triangles are a handy tool for remembering formulas like these. The speed one is shown below.

HOW DO YOU USE FORMULA TRIANGLES?
1) COVER UP the thing you want to find and WRITE DOWN what's left showing.
2) Now PUT IN THE VALUES and CALCULATE — check the UNITS in your answer.

EXAMPLE: A car travels 9 miles at 36 miles per hour. How many minutes does it take?

Write down the formula, put in the values and calculate: $\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{9 \text{ miles}}{36 \text{ mph}} = 0.25 \text{ hours} = 15 \text{ minutes}$

Knowledge Organiser: Mathematics
Year 11 Higher Spring Term 1

Suggested websites: Maths Genie, Save My Exam and Corbett Maths

Big idea: Ratio proportion and Rates of Change

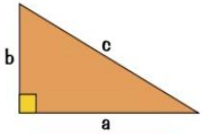
Key skills:
- Multiplicative Reasoning
- Geometric Reasoning
- Algebraic Reasoning

Key Vocabulary

Mass, Volume, Density, Pressure, Force, Area, Sine, Cosine, Tangent, Pythagoras Theorem

Pythagoras' Theorem — $a^2 + b^2 = c^2$ (Grade 4)

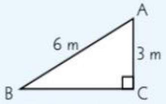
1) PYTHAGORAS' THEOREM only works for RIGHT-ANGLED TRIANGLES.
2) Pythagoras uses two sides to find the third side.
3) The BASIC FORMULA for Pythagoras is $a^2 + b^2 = c^2$
4) Make sure you get the numbers in the RIGHT PLACE. c is the longest side (called the hypotenuse) and it's always opposite the right angle.
5) Always CHECK that your answer is SENSIBLE.



$a^2 + b^2 = c^2$

EXAMPLE:

ABC is a right-angled triangle.
AB = 6 m and AC = 3 m.
Find the exact length of BC.



It's not always c you need to find — loads of people go wrong here.

1) Write down the formula. $a^2 + b^2 = c^2$
2) Put in the numbers. $BC^2 + 3^2 = 6^2$
3) Rearrange the equation. $BC^2 = 6^2 - 3^2 = 36 - 9 = 27$
4) Take square roots to find BC. $BC = \sqrt{27} = 3\sqrt{3}$ m
5) 'Exact length' means you should give your answer as a surd — simplified if possible.

Remember to check the answer's sensible — here it's about 5.2, which is between 3 and 6, so that seems about right.

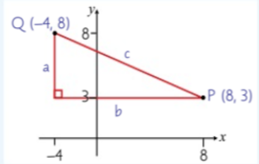
Use Pythagoras to find the Distance Between Points (Grade 5)

You need to know how to find the straight-line distance between two points on a graph.
If you get a question like this, follow these rules and it'll all become breathtakingly simple:

1) Draw a sketch to show the right-angled triangle.
2) Find the lengths of the shorter sides of the triangle by subtracting the coordinates.
3) Use Pythagoras to find the length of the hypotenuse. (That's your answer.)

EXAMPLE: Point P has coordinates (8, 3) and point Q has coordinates (–4, 8). Find the length of the line PQ.

①



② Length of side a = 8 – 3 = 5
Length of side b = 8 – –4 = 12

③ Use Pythagoras to find side c:
$c^2 = a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169$
So: $c = \sqrt{169} = 13$

Density = Mass ÷ Volume (Grade 4)

Density is the mass per unit volume of a substance. It's usually measured in kg/m³ or g/cm³.

$$\text{DENSITY} = \frac{\text{MASS}}{\text{VOLUME}} \qquad \text{VOLUME} = \frac{\text{MASS}}{\text{DENSITY}} \qquad \text{MASS} = \text{DENSITY} \times \text{VOLUME}$$

EXAMPLE: A giant 'Wunda-Choc' bar has a density of 1.3 g/cm³.
If the bar's volume is 1800 cm³, what is the mass of the bar in kg?

Write down the formula, put in the values and calculate:
mass = density × volume
= 1.3 g/cm³ × 1800 cm³ = 2340 g
= 2.34 kg

CHECK YOUR UNITS MATCH
If the density is in g/cm³, the volume must be in cm³ and you'll get a mass in g.

Pressure = Force ÷ Area (Grade 4)

'N' stands for 'Newtons'.

Pressure is the amount of force acting per unit area. It's usually measured in N/m², or pascals (Pa).

$$\text{PRESSURE} = \frac{\text{FORCE}}{\text{AREA}} \qquad \text{AREA} = \frac{\text{FORCE}}{\text{PRESSURE}} \qquad \text{FORCE} = \text{PRESSURE} \times \text{AREA}$$

EXAMPLE: A cylindrical barrel with a weight of 200 N rests on horizontal ground.
The radius of the circular face resting on the ground is 0.4 m.
Calculate the pressure exerted by the barrel on the ground to 1 d.p.

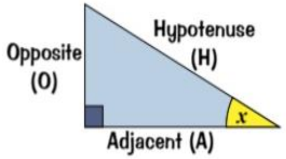
Work out the area of the circular face: $\pi \times 0.4^2 = 0.5026...$ m²
Write down the pressure formula, put in the values and calculate:
$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{200\text{ N}}{0.5026...\text{ m}^2} = 397.8873...\text{ N/m}^2$$
= 397.9 N/m² (1 d.p.)

The 3 Trigonometry Formulas (Grade 6)

There are three basic trig formulas — each one links two sides and an angle of a right-angled triangle.

$$\text{Sin } x = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cos } x = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{Tan } x = \frac{\text{Opposite}}{\text{Adjacent}}$$

- The Hypotenuse is the LONGEST SIDE.
- The Opposite is the side OPPOSITE the angle being used (x).
- The Adjacent is the (other) side NEXT TO the angle being used.



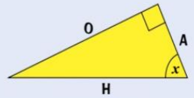
1) Whenever you come across a trig question, work out which two sides of the triangle are involved in that question — then pick the formula that involves those sides.
2) To find the angle — use the inverse, i.e. press SHIFT or 2ndF, followed by sin, cos or tan (and make sure your calculator is in DEG mode) — your calculator will display $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.
3) Remember, you can only use the sin, cos and tan formulas above on right-angled triangles — you may have to add lines to the diagram to create one.

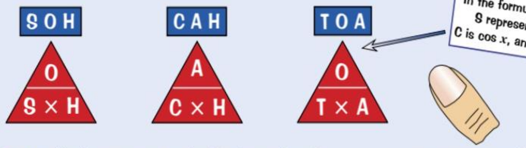
Formula Triangles Make Things Simple (Grade 6)

formula triangles on p69 if you need to jog your memory.

A handy way to tackle trig questions is to convert the formulas into formula triangles.
Then you can use the same method every time, no matter which side or angle is being asked for.

1) Label the three sides O, A and H (Opposite, Adjacent and Hypotenuse).
2) Write down from memory 'SOH CAH TOA'.
3) Decide which two sides are involved: O,H A,H or O,A and select SOH, CAH or TOA accordingly.
4) Turn the one you choose into a FORMULA TRIANGLE:





In the formula triangles, S represents sin x, C is cos x, and T is tan x.

5) Cover up the thing you want to find (with your finger), and write down whatever is left showing.
6) Translate into numbers and work it out.
7) Finally, check that your answer is sensible.

If you can't make SOH CAH TOA stick, try using a mnemonic like 'Strange Orange Hamsters Creep Around Houses Tripping Over Ants'.

Finding the nth Term of a Quadratic Sequence (Grade 7)

A quadratic sequence has an n² term — the difference between the terms changes as you go through the sequence, but the difference between the differences is the same each time.

EXAMPLE: Find an expression for the nth term of the sequence that starts 10, 14, 20, 28...

n: 1 2 3 4
term: 10 14 20 28
+4 +6 +8
+2 +2

So the expression will contain an n² term.

term:	10	14	20	28
n²:	1	4	9	16
term – n²:	9	10	11	12

The expression for this linear sequence is n + 8

So the expression for the nth term is $n^2 + n + 8$

1) Find the difference between each pair of terms.
2) The difference is changing, so work out the difference between the differences.
3) Divide this value by 2 — this gives the coefficient of the n² term (here it's 2 ÷ 2 = 1).
4) Subtract the n² term from each term in the sequence. This will give you a linear sequence.
5) Find the rule for the nth term of the linear sequence (see above) and add this on to the n² term.

Again, make sure you check your expression by putting the first few values of n back in — so n = 1 gives $1^2 + 1 + 8 = 10$, n = 2 gives $2^2 + 2 + 8 = 14$ and so on.